



Éducation des adultes
CENTRE FRÈRE-MOFFET

Exercices supplémentaires

Solutionnaire

MAT- 4106

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Février 2006

MAT-4106-1

EXERCICES SUPPLÉMENTAIRES

Corrigé

Dimension 1

1. $7x^2 - 14 = 7(x^2 - 2)$
2. $4a^2b^5 + 8a^2b^4 - 12a^3 = 4a^2(b^5 + 2b^4 - 3a)$
3. $2cx + bxc + cxy = cx(2 + b + y)$
4. $21bdxy + 49ady = 7dy(3bx + 7a)$
5. $a^8b^3 + a^7b^4 + a^6b^5 = a^6b^3(a^2 + ab + b^2)$
6. $16m^2 - 4m^2n - 8mn^2 = 4m(4m - mn - 2n^2)$
7. $10x^2 - 8x = 2x(5x - 4)$
8. $x^7 + x^6 + x^5 + x^4 = x^4(x^3 + x^2 + x + 1)$
9. $27x^2 - 45x + 54 = 9(3x^2 - 5x + 6)$
10. $r^3 + 2r^2s^2 + r^2 = r^2(r + 2s^2 + 1)$
11. $2(x + y) + a(x + y) = (x + y)(2 + a)$
12. $9m^4n^5o^2p^4 - 27o^3p^6q + 18m^6o^2pq^5 - 36mo^2p^5q^2 + 90o^3p^4q^6r^3 =$
Rép. : $9o^2p(m^4n^5p^3 - 3op^5q + 2m^6q^5 - 4mp^4q^2 + 10op^3q^6r^3)$

Dimension 2

1. $am - an - bm + bn = a(m - n) - b(m - n) = (m - n)(a - b)$
2. $10x^2 + 5x + 6x + 3 = 5x(2x + 1) + 3(2x + 1) = (2x + 1)(5x + 3)$
3. $3xy + 3xz + 2y^2 + 2yz = 3x(y + z) + 2y(y + z) = (y + z)(3x + 2y)$
4. $7xy + 14xz + y + 2z = 7x(y + 2z) + (y + 2z) = (y + 2z)(7x + 1)$
5. $4x^3 + 4x^2 - 4x - 4 = 4x^2(x + 1) - 4(x + 1) = (4x^2 - 4)(x + 1) = 4(x + 1)^2(x - 1)$
6. $5ac + 5bc - 10c + 8ad + 8bd - 16d = 5c(a + b - 2) + 8d(a + b - 2) = (a + b - 2)(5c + 8d)$
7. $2ax + 2ay - 2az + bx + by - bz = 2a(x + y - z) + b(x + y - z) = (x + y - z)(2a + b)$
8. $0,5x^2y + 1,5xy^2 - 3x^3y^2 - 9x^2y^3 = 0,5xy(x + 3y) - 3x^2y^2(x + 3y) =$
 $(0,5xy - 3x^2y^2)(x + 3y) = 0,5xy(x + 3y)(1 - 6xy)$
9. $2ax^3 - 2ax^2 + 8a - 3bx^3 - 3bx^2 - 12b = 2a(x^3 + x^2 + 4) - 3b(x^3 + x^2 + 4) = (x^3 + x^2 + 4)(2a - 3b)$
10. $6ax^2y^3 + 5z^5 - 10az^4 - 3x^2y^3z - 7xz^4 + 14axz^3 = -2a(-3x^2y^3 + 5z^4 - 7xz^3) + z(5z^4 - 3x^2y^3 - 7xz^3) =$
 $(z - 2a)(5z^4 - 3x^2y^3 - 7xz^3)$
11. $3x^3y^5 - 6x^4y^3 + 18x^5y^3 - 9x^4y^5 = 3x^3y^3(y^2 - 2x) - 9x^4y^3(-2x + y^2) = (y^2 - 2x)(3x^3y^3 - 9x^4y^3) =$
 $3x^3y^3(1 - 3x)(y^2 - 2x)$
12. $3ac + 3bc + ad + bd = 3c(a + b) + d(a + b) = (3c + d)(a + b)$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 3

1. $x^2 - 6x + 9 = (x - 3)(x - 3)$
2. $x^2 + 9xy + 14y^2 = (x + 2y)(x + 7y)$
3. $a^2 - 13a + 30 = (a - 3)(a - 10)$
4. $y^2 + 2y - 24 = (y + 6)(y - 4)$
5. $m^4 - 2m^2n - 3n^2 = (m^2 - 3n)(m^2 + n)$
6. $x^2 + 12x + 35 = (x + 5)(x + 7)$
7. $a^2 + 16 - 17a = a^2 - 17a + 16 = (a - 16)(a - 1)$
8. $x^4 - 2x^2y^2 - 48y^4 = (x^2 - 8y^2)(x^2 + 6y^2)$
9. $-a^2 - a + 20 = (-a + 4)(a + 5)$
10. $m^2 - 3mn^2 - 18n^4 = (m - 6n^2)(m + 3n^2)$
11. $x^4 - 16x^2y^2 + 55y^4 = (x^2 - 11y^2)(x^2 - 5y^2)$
12. $x^2 + 7x - 18 = (x + 9)(x - 2)$

Dimension 4

1. $12x^2 + 11xy + 2y^2 = (12x^2 + 3xy) + (8xy + 2y^2) = 3x(4x+y) + 2y(4x+y) = (4x + y)(3x + 2y)$
2. $6a^2 - ab - b^2 = (6a^2 - 3ab) + (2ab - b^2) = 3a(2a-b) + b(2a-b) = (2a - b)(3a + b)$
3. $27a^2 - 33ab + 10b^2 = (27a^2 - 15ab) - (18ab - b^2) = 3a(9a-5b) - 2b(9a-5b) = (9a - 5b)(3a - 2b)$
4. $-2a^2 + 5ab - 3b^2 = -(2a^2 - 2ab) + (3ab - 3b^2) = -2a(a - b) + 3b(a - b) = (a - b)(-2a + 3b)$
5. $42a^2 + 10b^2 - 47ab = 42a^2 - 47ab + 10b^2 = (42a^2 - 35ab) - (12ab - 10b^2) = 7a(6a - 5b) - 2b(6a - 5b) = (6a - 5b)(7a - 2b)$
6. $10s^2 + 17st + 3t^2 = (10s^2 + 15st) + (2st + 3t^2) = 5s(2s + 3t) + t(2s + 3t) = (2s + 3t)(5s + t)$
7. $72x^2 - y^2 + xy = (72x^2 + 9xy) - (8xy + y^2) = 9x(8x + y) - y(8x + y) = (8x + y)(9x - y)$
8. $22m^2 + 7mn - 2n^2 = (22m^2 + 11mn) - (4mn + 2n^2) = 11m(2m + n) - 2n(2m + n) = (2m + n)(11m - 2n)$
9. $51x^2 + 31xy - 2y^2 = (51x^2 + 34xy) - (3xy + 2y^2) = 17x(3x + 2y) - y(3x + 2y) = (3x + 2y)(17x - y)$
10. $8x^2 - 34xy + 21y^2 = (8x^2 - 28xy) - (6xy - 21y^2) = 4x(2x - 7y) - 3y(2x - 7y) = (2x - 7y)(4x - 3y)$
11. $3a^2 - 25ab + 52b^2 = (3a^2 - 13ab) - (12ab - 52b^2) = a(3a - 13b) - 4b(3a - 13b) = (3a - 13b)(a - 4b)$
12. $4c^2 + 28cd + 49d^2 = (4c^2 + 14cd) + (14cd + 49d^2) = 2c(2c + 7d) + 7d(2c + 7d) = (2c + 7d)^2$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 5

- $y^2 - 49 = (y - 7)(y + 7)$
- $64c^6 - 49d^4e^4 = (8c^3 - 7d^2e^2)(8c^3 + 7d^2e^2)$
- $4a^2b^4c^6 - 9d^4 = (2ab^2c^3 - 3d^2)(2ab^2c^3 + 3d^2)$
- $25x^4 - 100y^2 = (5x^2 - 10y)(5x^2 + 10y)$
- $16 - m^4n^8 = (2 - mn^2)(2 + mn^2)(4 + m^2n^4)$
- $25y^6 - 100x^8y^8 = 25y^6(1 - 4x^8y^2) = 25y^6(1 - 2x^4y)(1 + 2x^4y)$
- $49m^2/100 - 25 = (7m/10 - 5)(7m/10 + 5)$
- $25x^4/36 - 9y^4z^8/16 = (5x^2/6 - 3y^2z^4/4)(5x^2/6 + 3y^2z^4/4)$
- $9q^4/16 - 25/36 = (3q^2/4 - 5/6)(3q^2/4 + 5/6)$
- $400 - 169p^8/900 = (20 - 13p^4/30)(20 + 13p^4/30)$
- $4y^8/25 - 0,64 = (2y^4/5 - 0,8)(2y^4/5 + 0,8)$
- $-9q^4/16 + 25r^2/36 = (-3q^2/4 + 5r/6)(3q^2/4 + 5r/6)$

Dimension 6

- $c^5 + c^4d - cd^2 - d^3 = c^4(c + d) - d^2(c + d) = (c + d)(c^4 - d^2) = (c^2 - d)(c^2 + d)(c + d)$
- $4a^3 - 16a^2b - 9ab^2 + 36b^3 = 4a^2(a - 4b) - 9b^2(a - 4b) = (a - 4b)(4a^2 - 9b^2) = (a - 4b)(2a - 3b)(2a + 3b)$
- $m^4/81 - n^4/16 = (m^2/9 - n^2/4)(m^2/9 + n^2/4) = (m/3 - n/2)(m/3 + n/2)(m^2/9 + n^2/4)$
- $8a^3 - 4a^2 - 2a + 1 = 4a^2(2a - 1) - (2a - 1) = (4a^2 - 1)(2a - 1) = (2a - 1)(2a + 1)(2a - 1)$
ou $(2a - 1)^2(2a + 1)$
- $16a^4b - 10a^6c^4 - 8a^3b^2 + 5a^5bc^4 = 2a^4(8b - 5a^2c^4) - a^3b(8b - 5a^2c^4) = (8b - 5a^2c^4)(2a^4 - a^3b) = a^3(2a - b)(8b - 5a^2c^4)$
- $6a^2b + 30ab - 12ab^2 - 60b^2 = 6ab(a + 5) - 12b^2(a + 5) = (a + 5)(6ab - 12b^2) = 6b(a - 2b)(a + 5)$
- $2a^2 - 8b^2 = 2(a^2 - 4b^2) = 2(a - 2b)(a + 2b)$
- $5abx - 5axy + 5bx^2 - 5x^2y = 5ax(b - y) + 5x^2(b - y) = (5ax + 5x^2)(b - y) = 5x(a + x)(b - y)$
- $100x^4 - 36a^4b^2 = 4(25x^4 - 9a^4b^2) = 4(5x^2 - 3a^2b)(5x^2 + 3a^2b)$
- $9a^6 - 36a^4b^2 = 9(a^6 - 4a^4b^2) = 9a^4(a - 2b)(a + 2b)$
- $s^3 - s = s(s^2 - 1) = s(s - 1)(s + 1)$
- $x^4 - a^4 = (x^2 - a^2)(x^2 + a^2) = (x - a)(x + a)(x^2 + a^2)$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 7

1. $30s^3 - 25s^2t^2 - 30st^4 = 5s(6s^2 - 5st^2 - 6t^4) = 5s[(6s^2+9st^2)-(4st^2+6t^4)] = 5s[3s(2s+3t^2) - 2t^2(2s+3t^2)] = 5s(2s+3t^2)(3s-2t^2)$
2. $45pq + 6p^2q^2 - 3p^3q^3 = 3pq(15 + 2pq - p^2q^2) = 3pq(5 - pq)(3 + pq)$
3. $5st^2 + 32s^2t^2 - 21s^3t^2 = st^2(5+32s-21s^2) = st^2[(5+35s)-(3s+21s^2)] = st^2[5(1+7s)-3s(1+7s)] = st^2(5 - 3s)(1 + 7s)$
4. $3a^2b - 9ab - 30b = 3b(a^2 - 3a - 10) = 3b(a - 5)(a + 2)$
5. $-33m^3 + 69m^2 - 6m = m(33m^2 - 69m + 6) = m[(33m^2 - 66m) - (3m - 6)] = [33m(m - 2) - 3(m - 2)] = m(33m - 3)(m - 2)$
6. $-4m^5 - 4m^3 + 24m = -4m(m^4 + m^2 - 6) = -4m(m^2 + 3)(m^2 - 2)$
7. $15x^2y^2 + 95xy^2 - 70y^2 = 5y^2(3x^2 + 19x - 14) = 5y^2[(3x^2 = 21x) - (2x + 14)] = 5y^2[3x(x + 7) - 2(x + 7)] = 5y^2(3x - 2)(x + 7)$
8. $a^2x + 2abx + b^2x = x(a^2 + 2ab + b^2) = x(a + b)(a + b)$
9. $a^2t^2 - 16a^2t + 39a^2 = a^2(t^2 - 16t + 39) = a^2(t - 3)(t - 13)$
10. $6c^2 + 6cd - 12d^2 = 6(c^2 + cd - 2d^2) = 6(c + 2d)(c - d)$
11. $11x^2 - 42x + x^3 = x(11x - 42 + x^2) = x(x^2 + 11x - 42) = x(x + 14)(x - 3)$
12. $8x^3 + 32x^2 + 30x = 2x(4x^2 + 16x + 15) = 2x[(4x^2 + 10) + (6x + 5)] = 2x[2x(2x + 5) + 3(2x + 5)] = 2x(2x + 3)(2x + 5)$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 8

$$1. \frac{x^2 + 4x - 21}{x^2 + 13x - 48} = \frac{(x+7)(\cancel{x-3})}{(\cancel{x-3})(x+16)} = \frac{(x+7)}{(x+16)}$$

$$2. \frac{a^2 - 1}{2a^2 - a - 3} = \frac{(a-1)(\cancel{a+1})}{(\cancel{a+1})(2a-3)} = \frac{(a-1)}{(2a-3)}$$

$$3. \frac{24dx^2 - 18dx}{-2dx} = \frac{\cancel{6dx}(4x-3)}{\cancel{-2dx}} = \frac{-3}{-2} (4x-3) = \frac{3}{2} (4x-3)$$

$$4. \frac{cd - d - c + 1}{3cd - 3c} = \frac{(\cancel{d-1})(c-1)}{3c(\cancel{d-1})} = \frac{(c-1)}{3c}$$

$$5. \frac{-4a + 8b}{2b^2 + 3ab - 2a^2} = \frac{4(\cancel{-a+2b})}{(b+2a)(\cancel{2b-a})} = \frac{4}{(b+2a)}$$

$$6. \frac{10m^3 + 15m^2 - 10m}{2m^2 - 5m + 2} = \frac{5m(\cancel{2m-1})(m+2)}{(\cancel{2m-1})(m-2)} = \frac{5m(m+2)}{(m-2)}$$

$$7. \frac{9 + 12xy - 5x^2y^2}{3x^2y^2 - 10xy + 3} = \frac{-(-3+xy)(3+5xy)}{(3xy-1)(xy-3)} = \frac{-(3+5xy)}{(3xy-1)}$$

$$8. \frac{b^4 - a^4}{a^4 - 2a^2b^2 + b^4} = \frac{-(-b^2+a^2)(b^2+a^2)}{(a^2-b^2)(\cancel{a^2-b^2})} = \frac{-(b^2+a^2)}{(a^2-b^2)} \text{ ou } \frac{-(a^2+b^2)}{(a-b)(a+b)}$$

$$9. \frac{2x^4 - 5x^2y^2 + 3y^4}{2x^4 - x^2y^2 - 3y^4} = \frac{(2x^2-3y^2)(x^2-y^2)}{(2x^2-3y^2)(x^2+y^2)} = \frac{(x^2-y^2)}{(x^2+y^2)} \text{ ou } \frac{(x-y)(x+y)}{(x^2+y^2)}$$

$$10. \frac{12a^2 - 5ab - 2b^2}{9a^2 - 12ab + 4b^2} = \frac{(4a+b)(\cancel{3a-2b})}{(3a-2b)(\cancel{3a-2b})} = \frac{(4a+b)}{(3a-2b)}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 9

$$1. \frac{3x+5}{4(x-1)} \times \frac{x-1}{3x+5} = \frac{1}{4}$$

$$2. \frac{a^2-1}{a^3-a^2} \times \frac{8a^4}{a+1} = \frac{(a-1)(a+1)}{a^2(a-1)} \times \frac{8a^4}{(a+1)} = 8a^2$$

$$3. \frac{4a+a^2}{a^3} \times \frac{12-3a}{16-a^2} = \frac{a(4+a)}{a^3 a^2} \times \frac{3(4-a)}{(4-a)(4+a)} = \frac{3}{a^2}$$

$$4. \frac{d^2-2d}{8-2d-d^2} \times \frac{3d+12}{6d} = \frac{-d(-d+2)}{(4+d)(2-d)} \times \frac{3(d+4)}{2 \cdot 6d} = \frac{-1}{2}$$

$$5. \frac{a^2-a-6}{16a^2} \times \frac{8a^3-24a^2}{a+2} = \frac{(a-3)(a+2)}{2 \cdot 16a^2} \times \frac{8a^2(a-3)}{(a+2)} = \frac{(a-3)^2}{2}$$

$$6. \frac{30ax^2+10axy-15bx^2-5bxy}{-3x-y} \times \frac{a^2+b^2}{40a^2x-10b^2x} = \frac{5x(2a-b)(3x+y)}{-(3x+y) \times 10x(2a-b)(2a+b)} \times \frac{(a^2+b^2)}{2} = \frac{-(a^2+b^2)}{2(2a+b)}$$

$$7. \frac{16a-a^3}{12+7a+a^2} \times \frac{3a^3+a^4}{a^4-4a^3} = \frac{-a(4+a)(4+a)}{(3+a)(4+a)} \times \frac{a^3(3+a)}{a^3(a-4)} = -a$$

$$8. \frac{4xy^2-12y}{2x^2+7x+3} \times \frac{2x+1}{xy^2-3y} = \frac{4y(xy-3)}{(x+3)(2x+1)} \times \frac{(2x+1)}{y(xy-3)} = \frac{4}{(x+3)}$$

$$9. \frac{x^2+3xy-18y^2}{-2x^2-10x+12} \times \frac{4x^2+4x-8}{x^2-2xy-3y^2} = \frac{(x+6y)(x-3y)}{-2(x+6)(x-1)} \times \frac{4^2(x+2)(x-1)}{(x-3y)(x+y)} = \frac{-2(x+6y)(x+2)}{(x+6)(x+y)}$$

$$10. \frac{8}{x+1} \times \frac{x^2-3x-4}{8x+4y} = \frac{2}{(x+1)} \times \frac{(x-4)(x+1)}{4(2x+y)} = \frac{2(x-4)}{(2x+y)}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 10

- $$1. \frac{a^2 - 16}{a^3} \div \frac{a^2 - 5a + 4}{3a^3 - 3a^2} = \frac{(a-4)(a+4)}{a^3} \times \frac{3a^2(a-1)}{(a-4)(a-1)} = \frac{3(a+4)}{a}$$
- $$2. \frac{m^2 - 49}{(m+7)^2} \div \frac{3m-21}{2m+14} = \frac{(m-7)(m+7)}{(m+7)(m+7)} \times \frac{2(m+7)}{3(m-7)} = \frac{2}{3}$$
- $$3. \frac{2p^2 - 5p + 2}{2p^2} \div (5p - 2 - 2p^2) = \frac{(p-2)(2p-1)}{2p^2} \times \frac{1}{-(p-2)(2p-1)} = \frac{-1}{2p^2}$$
- $$4. \frac{d^2 - 3df - 10f^2}{d^2 - 25f^2} \div \frac{10d + 20f}{2d + 10f} = \frac{(d-5f)(d+2f)}{(d-5f)(d+5f)} \times \frac{2(d+5f)}{10(d+2f)} = \frac{1}{5}$$
- $$5. \frac{x^4 - 1}{x^2 + 1 - 2x} \div \frac{2x^2 + 2x}{x^2 - x} = \frac{(x^2 - 1)(x^2 + 1)}{(x-1)(x+1)} \times \frac{x(x-1)}{2x(x+1)} = \frac{(x-1)(x+1)(x^2 + 1)}{2(x-1)(x+1)} = \frac{(x^2 + 1)}{2}$$
- $$6. \frac{y^2 - x^2}{x^2 + 2xy + y^2} \div \frac{x^2 - 2xy + y^2}{x^2 + xy} = \frac{-(-y+x)(y+x)}{(x+y)(x+y)} \times \frac{x(x+y)}{(x-y)(x-y)} = \frac{-x}{(x-y)}$$
- $$7. \frac{a^2 + 7a - 30}{a^4 + 3a^3} \div \frac{a^4 - 3a^3}{-a^6} = \frac{(a+10)(a-3)}{a^3(a+3)} \times \frac{-a^6}{a^3(a-3)} = \frac{-(a+10)}{(a+3)}$$
- $$8. \frac{4x - 2y}{(2x - y)^2} \div \frac{12x + 6y}{4x^2 - y^2} = \frac{-2(2x-y)}{(2x-y)(2x-y)} \times \frac{(2x-y)(2x+y)}{-6(2x+y)} = \frac{1}{3}$$
- $$9. \frac{2x^2 + 3x - 5}{2x^2 - 7x - 30} \div \frac{1 - x^2}{-4x + 24} = \frac{(2x+5)(x-1)}{(2x+5)(x-6)} \times \frac{-4(x-6)}{-(-1+x)(1+x)} = \frac{4}{(1+x)}$$
- $$10. \frac{4c^2 - d^2}{6c^2 + 5cd + d^2} \div \frac{d^2 - 4c^2}{9c^2 - d^2} = \frac{-(-2c+d)(2c+d)}{(2c+d)(3c+d)} \times \frac{(3c+d)(3c-d)}{(d-2c)(d+2c)} = \frac{-(3c-d)}{(d+2c)}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 11

$$1. \frac{2x}{a-b} + \frac{3y}{a-b} = \frac{2x+3y}{a-b}$$

$$2. \frac{m-n}{2n} + \frac{m+n}{3m} = \frac{3m(m-n) + 2n(m+n)}{6mn} = \frac{3m^2 - 3mn + 2mn + 2n^2}{6mn} = \frac{3m^2 - mn + 2n^2}{6mn}$$

$$3. \frac{3t}{2s+t} + \frac{2s-t}{s} = \frac{3st + (2s+t)(2s-t)}{s(2s+t)} = \frac{3st + 4s^2 - t^2}{s(2s+t)} = \frac{4s^2 + 3st - t^2}{s(2s+t)} = \frac{4s^2 + 4st - st + t^2}{s(2s+t)}$$
$$= \frac{4s(s+t) - t(s+t)}{s(2s+t)} = \frac{(s+t)(4s-t)}{s(2s+t)}$$

$$4. \frac{c+d}{3c+3d} + \frac{2c}{d-2c} = \frac{e+d}{3(e+d)} + \frac{2c}{d-2c} = \frac{d-2c+6d}{3(d-2c)} = \frac{d+4c}{3(d-2c)}$$

$$5. \frac{-3c+6d}{2c(c-2d)} + \frac{6d}{2cd+4d^2} = \frac{-3(e-2d)}{2c(e-2d)} + \frac{3}{2d(c+2d)} = \frac{-3}{2c} + \frac{3}{c+2d} = \frac{-3(c+2d)+6c}{2c(c+2d)}$$
$$= \frac{-3c-6d+6c}{2c(c+2d)} = \frac{3c-6d}{2c(c+2d)} = \frac{3(c-2d)}{2c(c+2d)}$$

$$6. \frac{a^2-2ab+b^2}{a^2-b^2} + \frac{2ab}{2a^2+3ab+b^2} = \frac{(a-b)(a-b)}{(a-b)(a+b)} + \frac{2ab}{(a+b)(2a+b)} = \frac{a-b}{a+b} + \frac{2ab}{(a+b)(2a+b)}$$
$$= \frac{(2a+b)(a-b) + 2ab}{(a+b)(2a+b)} = \frac{2a^2 - 2ab + ab - b^2 + 2ab}{(a+b)(2a+b)} = \frac{2a^2 + ab - b^2}{(a+b)(2a+b)} = \frac{(2a-b)(a+b)}{(a+b)(2a+b)} = \frac{2a-b}{2a+b}$$

$$7. \frac{4x^2-4xy+y^2}{2x^2+xy-y^2} + \frac{3x^2-3xy}{2y(x+y)} = \frac{(2x-y)(2x-y)}{(2x-y)(x+y)} + \frac{3x(x-y)}{2y(x+y)} = \frac{2y(2x-y) + 3x(x-y)}{2y(x+y)}$$
$$= \frac{4xy - 2y^2 + 3x^2 - 3xy}{2y(x+y)} = \frac{3x^2 + xy - 2y^2}{2y(x+y)} = \frac{(x+y)(3x-2y)}{2y(x+y)} = \frac{3x-2y}{2y}$$

$$8. \frac{-(x-5)}{3x^2+5x-2} + \frac{x+6}{9x^2-1} = \frac{-(x-5)}{(x+2)(3x-1)} + \frac{x+6}{(3x-1)(3x+1)} = \frac{-(x+5)(3x+1) + (x+6)(x+2)}{(x+2)(3x-1)(3x+1)}$$
$$= \frac{-3x^2 - x + 15x + 5 + x^2 + 8x + 12}{(x+2)(3x-1)(3x+1)} = \frac{-2x^2 + 22x + 17}{(x+2)(3x-1)(3x+1)}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 11 (suite)

$$9. \frac{x^2 + 2x + 20}{x^2 + 4x - 12} + \frac{x + 7}{x + 6} = \frac{x^2 + 2x + 20}{(x + 6)(x - 2)} + \frac{x + 7}{x + 6} = \frac{x^2 + 2x + 20}{(x + 6)(x - 2)} + \frac{x^2 - 2x + 7x - 14}{(x + 6)(x - 2)} =$$
$$= \frac{2x^2 + 7x + 6}{(x + 6)(x - 2)} = \frac{(x + 2)(2x + 3)}{(x + 6)(x - 2)}$$

$$10. \frac{x + 5}{x^2 + 11x + 30} + \frac{x + 16}{x^2 + 2x - 24} = \frac{(x + 5)}{(x + 6)(x + 5)} + \frac{x + 16}{(x + 6)(x - 4)} = \frac{x - 4 + x + 16}{(x + 6)(x - 4)} =$$
$$= \frac{2x + 12}{(x + 6)(x - 4)} = \frac{2(x + 6)}{(x + 6)(x - 4)} = \frac{2}{x - 4}$$

Dimension 12

$$1. \frac{m - 2n}{m} - \frac{2n}{m} = \frac{m - 2n - 2n}{m} = \frac{m - 4n}{m}$$

$$2. \frac{6}{x - y} - \frac{4}{x + y} = \frac{6(x + y) - 4(x - y)}{(x - y)(x + y)} = \frac{6x + 6y - 4x + 4y}{(x - y)(x + y)} = \frac{2x + 10y}{(x - y)(x + y)} = \frac{2(x + 5y)}{(x - y)(x + y)}$$

$$3. \frac{5m}{2m + 3} - \frac{3m}{3m - 2} = \frac{5m(3m - 2) - 3m(2m + 3)}{(2m + 3)(3m - 2)} = \frac{15m^2 - 10m - 6m^2 - 9m}{(2m + 3)(3m - 2)} =$$
$$= \frac{9m^2 - 19m}{(2m + 3)(3m - 2)} = \frac{m(9m - 19)}{(2m + 3)(3m - 2)}$$

$$4. \frac{12mn - 6n^2}{2m^3 - m^2n} - \frac{m - n}{m^2 + mn} = \frac{6n(2m - n)}{m^2(2m - n)} - \frac{m - n}{m(m + n)} = \frac{6n(m + n) - m(m - n)}{m^2(m + n)}$$
$$= \frac{6mn + 6n^2 - m^2 + mn}{m^2(m + n)} = \frac{-m^2 + 7mn + 6n^2}{m^2(m + n)}$$

$$5. \frac{2x}{x - y} - \frac{x^2 - y^2}{x^2 - 2xy + y^2} = \frac{2x}{x - y} - \frac{(x - y)(x + y)}{(x - y)(x - y)} = \frac{2x - x - y}{(x - y)} = \frac{x - y}{(x - y)} = 1$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 12(suite)

$$\begin{aligned} 6. \frac{x^2 + 2xy}{x^2 - 3xy} - \frac{3y}{x - y} &= \frac{\cancel{x}(x + 2y)}{\cancel{x}(x - 3y)} - \frac{3y}{x - y} = \frac{(x + 2y)(x - y) - 3y(x - 3y)}{(x - 3y)(x - y)} = \\ &= \frac{x^2 - xy + 2xy - 2y^2 - (3xy + 9y^2)}{(x - 3y)(x - y)} = \frac{\mathbf{x^2 - 2xy + 7y^2}}{(x - 3y)(x - y)} \end{aligned}$$

$$\begin{aligned} 7. \frac{x - 3}{5x + 30} - \frac{x + 3}{5x + 10} &= \frac{x - 3}{5(x + 6)} - \frac{x + 3}{5(x + 2)} = \frac{(x + 2)(x - 3) - (x + 6)(x + 3)}{5(x + 6)(x + 2)} = \\ &= \frac{x^2 - 3x + 2x - 6 - x^2 - 3x - 6x - 18}{5(x + 6)(x + 2)} = \frac{-10x - 24}{5(x + 6)(x + 2)} = \frac{\mathbf{-2(5x + 12)}}{\mathbf{5(x + 6)(x + 2)}} \end{aligned}$$

$$\begin{aligned} 8. \frac{x + 1}{x - 4} - \frac{x - 5}{x + 5} &= \frac{(x + 1)(x + 5) - (x - 4)(x - 5)}{(x - 4)(x + 5)} = \frac{x^2 + 6x + 5 - x^2 + 9x - 20}{(x - 4)(x + 5)} = \\ &= \frac{15x - 15}{(x - 4)(x + 5)} = \frac{\mathbf{15(x - 1)}}{\mathbf{(x - 4)(x + 5)}} \end{aligned}$$

$$9. \frac{x}{x^2 - 49} - \frac{1}{x + 7} = \frac{x}{(x - 7)(x + 7)} - \frac{1}{x + 7} = \frac{x - (x - 7)}{(x - 7)(x + 7)} = \frac{\mathbf{7}}{\mathbf{(x - 7)(x + 7)}}$$

$$\begin{aligned} 10. \frac{x - 1}{x^2 + 12x + 32} - \frac{2x - 2}{x^2 + 8x} &= \frac{x - 1}{(x + 4)(x + 8)} - \frac{2(x - 1)}{x(x + 8)} = \frac{x(x - 1) - 2(x - 1)(x + 4)}{x(x + 4)(x + 8)} = \\ &= \frac{x^2 - x - 2(x^2 + 3x - 4)}{x(x + 4)(x + 8)} = \frac{x^2 - x - 2x^2 - 6x + 8}{x(x + 4)(x + 8)} = \frac{-x^2 - 7x + 8}{x(x + 4)(x + 8)} = \frac{(-x + 1)(x + 8)}{x(x + 4)(x + 8)} = \\ &= \frac{\mathbf{-x + 1}}{\mathbf{x(x + 4)}} \text{ ou } \frac{\mathbf{-(x - 1)}}{\mathbf{x(x + 4)}} \end{aligned}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 13

$$1. \frac{x+y}{x^2-y^2} + \frac{4x+4y}{x^2-2xy+y^2} = \frac{5x+3y}{(x-y)^2}$$

$$\frac{x+y}{(x+y)(x-y)} + \frac{4(x+y)}{(x-y)(x-y)} = \frac{5x+3y}{(x-y)^2}$$

$$\frac{x-y}{(x-y)(x-y)} + \frac{4(x+y)}{(x-y)^2} = \frac{5x+3y}{(x-y)^2}$$

$$\frac{x-y+4x+4y}{(x-y)^2} = \frac{5x+3y}{(x-y)^2}$$

$$2. \frac{x^2+4}{x^3-4x} - \frac{x+2}{2x^2-4x} = \frac{x^3-x^2-4}{2x(x+2)(x-2)}$$

$$\frac{x^2+4}{x(x^2-4)} - \frac{x+2}{2x(x-2)} = \frac{x^3-x^2-4}{2x(x+2)(x-2)}$$

$$\frac{x^2+4}{x(x-2)(x+2)} - \frac{x+2}{2x(x-2)} = \frac{x^3-x^2-4}{2x(x+2)(x-2)}$$

$$\frac{2(x^2+4) - (x+2)(x+2)}{2x(x+2)(x-2)} = \frac{2x^2+8 - (x^2+4x+4)}{2x(x+2)(x-2)}$$

$$\frac{x^2-4x+4}{2x(x+2)(x-2)} = \frac{x-2}{2x(x+2)}$$

$$3. \frac{x^2+2xy+y^2}{x^2-y^2} + \frac{x^2-y^2}{x^2+2xy+y^2} = \frac{2(x^2+y^2)}{x^2-y^2}$$

$$\frac{(x+y)^2}{(x-y)(x+y)} + \frac{(x-y)(x+y)}{(x+y)^2} = \frac{2(x^2+y^2)}{x^2-y^2}$$

$$\frac{(x+y)(x+y) + (x-y)(x-y)}{(x-y)(x+y)} = \frac{2(x^2+y^2)}{x^2-y^2}$$

$$\frac{x^2+2xy+y^2 + x^2-2xy+y^2}{(x-y)(x+y)} = \frac{2(x^2+y^2)}{x^2-y^2}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 13 (suite)

$$4. \frac{x-y}{2x^2+2xy} + \frac{y}{x^2-y^2} = \frac{x^2+y^2}{2x(x^2-y^2)}$$

$$\frac{x-y}{2x(x+y)} + \frac{y}{(x-y)(x+y)} = \frac{x^2+y^2}{2x(x^2-y^2)}$$

$$\frac{(x-y)(x-y) + 2xy}{2x(x+y)(x-y)} = \frac{x^2+y^2}{2x(x^2-y^2)}$$

$$\frac{x^2 - 2xy + y^2 + 2xy}{2x(x+y)(x-y)} = \frac{x^2+y^2}{2x(x^2-y^2)}$$

$$\frac{x^2+y^2}{2x(x+y)(x-y)} = \frac{x^2+y^2}{2x(x+y)(x-y)}$$

$$5. \frac{2a^2 - 2ab + b^2}{a^2 - ab} + \frac{a+b}{a} = \frac{3a-2b}{a-b}$$

$$\frac{2a^2 - 2ab + b^2}{a(a-b)} + \frac{a+b}{a} = \frac{3a-2b}{a-b}$$

$$\frac{2a^2 - 2ab + b^2 + (a+b)(a-b)}{a(a-b)} = \frac{3a-2b}{a-b}$$

$$\frac{2a^2 - 2ab + b^2 + a^2 - b^2}{a(a-b)} = \frac{3a-2b}{a-b}$$

$$\frac{3a^2 - 2ab}{a(a-b)} = \frac{\cancel{a}(3a-2b)}{\cancel{a}(a-b)} = \frac{3a-2b}{a-b}$$

$$6. \frac{ax-a}{ax^2-a} - \frac{x^2-1}{x^2-2x+1} = \frac{-(x^2+x+2)}{(x+1)(x-1)}$$

$$\frac{a(x-1)}{a(x^2-1)} - \frac{\cancel{(x-1)}(x+1)}{(x-1)\cancel{(x-1)}} = \frac{-(x^2+x+2)}{(x+1)(x-1)}$$

$$\frac{\cancel{a}\cancel{(x-1)}^1}{\cancel{a}\cancel{(x-1)}(x+1)} - \frac{(x+1)}{(x-1)} = \frac{-(x^2+x+2)}{(x+1)(x-1)}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 13 (suite)

$$\frac{1}{(x+1)} - \frac{(x+1)}{(x-1)} = \frac{-(x^2+x+2)}{(x+1)(x-1)}$$

$$\frac{x-1}{(x+1)(x-1)} - \frac{(x+1)(x+1)}{(x+1)(x-1)} = \frac{-(x^2+x+2)}{(x+1)(x-1)}$$

$$\frac{x-1-x^2-2x-2}{(x+1)(x-1)} = \frac{-(x^2+x+2)}{(x+1)(x-1)}$$

$$7. \frac{27b^2 - 48c^2}{6ab - 8ac} - \frac{3c(48b + 32c)}{12ab + 16ac} = \frac{27b^2}{6ab + 8ac}$$

$$\frac{3(9b^2 - 16c^2)}{2a(3b - 4c)} - \frac{48c(3b + 2c)}{4a(3b + 4c)} = \frac{27b^2}{6ab + 8ac}$$

$$\frac{3(3b - 4c)(3b + 4c)}{2a(3b - 4c)} - \frac{12c(3b + 2c)}{a(3b + 4c)} = \frac{27b^2}{6ab + 8ac}$$

$$\frac{3(3b + 4c)(3b + 4c) - 24c(3b + 2c)}{2a(3b + 4c)} = \frac{27b^2}{6ab + 8ac}$$

$$\frac{27b^2 + 72bc + 48c^2 - 72bc - 48c^2}{2a(3b + 4c)} = \frac{27b^2}{6ab + 8ac}$$

$$8. \frac{5a - b}{10a^2 + 13ab - 3b^2} + \frac{-3a + 3b}{4a^2 + 12ab + 9b^2} = - \frac{(a - 6b)}{(2a + 3b)^2}$$

$$\frac{5a - b}{(5a - b)(2a + 3b)} + \frac{-3(a - b)}{(2a + 3b)(2a + 3b)} = - \frac{(a - 6b)}{(2a + 3b)^2}$$

$$\frac{2a + 3b - 3(a - b)}{(2a + 3b)(2a + 3b)} = - \frac{(a - 6b)}{(2a + 3b)^2}$$

$$\frac{2a + 3b - 3a + 3b}{(2a + 3b)(2a + 3b)} = - \frac{(a - 6b)}{(2a + 3b)^2}$$

$$\frac{-a + 6b}{(2a + 3b)^2} = - \frac{(a - 6b)}{(2a + 3b)^2}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 14

1. $\frac{x}{x-4} + \frac{2}{x-9} = \frac{x^2 + 2x}{x^2 - 2x - 8} + \frac{2x + 6}{x^2 - 6x - 27}$

$$\frac{x(x-9) + 2(x-4)}{(x-4)(x-9)} = \frac{x(x+2)}{(x-4)(x+2)} + \frac{2(x+3)}{(x-9)(x+3)}$$

$$\frac{x^2 - 9x + 2x - 8}{(x-4)(x-9)} = \frac{x(x-9) + 2(x-4)}{(x-4)(x-9)}$$

$$\frac{x^2 - 7x - 8}{(x-4)(x-9)} = \frac{x^2 - 9x + 2x - 8}{(x-4)(x-9)}$$

$$\frac{(x-8)(x+1)}{(x-4)(x-9)} = \frac{(x-8)(x+1)}{(x-4)(x-9)}$$

2. $\frac{a^2 + b^2}{a^2b + ab^2} + \frac{a-b}{ab} = \frac{a^2 - b^2}{a^2b + 2ab^2 + b^3} + \frac{1}{b}$

$$\frac{a^2 + b^2}{ab(a+b)} + \frac{a-b}{ab} = \frac{(a-b)(\cancel{a+b})}{b(a+b)(\cancel{a+b})} + \frac{1}{b}$$

$$\frac{a^2 + b^2 + (a-b)(a+b)}{ab(a+b)} = \frac{(a-b)}{b(a+b)} + \frac{1}{b}$$

$$\frac{a^2 + b^2 + a^2 - b^2}{ab(a+b)} = \frac{a-b+a+b}{b(a+b)}$$

$$\frac{\cancel{2a^2}}{a \cdot b(a+b)} = \frac{2a}{b(a+b)}$$

3. $\frac{x^2 + 5x - 6}{x^2 + 2x - 24} - \frac{2x + 12}{x^2 + 7x + 6} = \frac{x^2 + x - 2}{x^2 - 2x - 8} - \frac{6x + 18}{3x^2 + 12x + 9}$

$$\frac{(x+6)(x-1)}{(x+6)(x-4)} - \frac{2(x+6)}{(x+6)(x+1)} = \frac{(x+2)(x-1)}{(x-4)(x+2)} - \frac{6(x+3)}{3(x+3)(x+1)}$$

$$\frac{x-1}{x-4} - \frac{2}{x+1} = \frac{x-1}{x-4} - \frac{2}{x+1}$$

EXERCICES SUPPLÉMENTAIRES (MAT-4106-1) Corrigé

Dimension 14(suite)

$$4. \frac{x+4}{x^2+2x} - \frac{2x+3}{x^3+2x^2} = \frac{x^2+3x-4}{x^3+x^2-2x} - \frac{2x^2+x-3}{x^4+x^3-2x^2}$$

$$\frac{x+4}{x(x+2)} - \frac{2x+3}{x^2(x+2)} = \frac{(x+4)(\cancel{x+1})}{x(x+2)(\cancel{x+1})} - \frac{(\cancel{x+1})(2x+3)}{x^2(x+2)(\cancel{x+1})}$$

$$\frac{x+4}{x(x+2)} - \frac{2x+3}{x^2(x+2)} = \frac{x+4}{x(x+2)} - \frac{2x+3}{x^2(x+2)}$$

$$5. \frac{x^2+3x+2}{x^2-2x-3} - \frac{x+3}{x+1} = \frac{2x^2+13x+35}{3x^2-6x-9} - \frac{2x+2}{3x-9}$$

$$\frac{(x+2)(\cancel{x+1})}{(x-3)(\cancel{x+1})} - \frac{x+3}{x+1} = \frac{2x^2+13x+35}{3(x-3)(x+1)} - \frac{2(x+1)}{3(x-3)}$$

$$\frac{(x+2)(x+1) - (x+3)(x-3)}{(x-3)(x+1)} = \frac{2x^2+13x+35 - 2(x+1)(x+1)}{3(x-3)(x+1)}$$

$$\frac{x^2+3x+2 - x^2+9}{(x-3)(x+1)} = \frac{2x^2+13x+35 - 2x^2 - 4x - 2}{3(x-3)(x+1)}$$

$$\frac{3x+11}{(x-3)(x+1)} = \frac{9x+33}{3(x-3)(x+1)}$$

$$\frac{3x+11}{(x-3)(x+1)} = \frac{\cancel{3}(3x+11)}{\cancel{3}(x-3)(x+1)}$$

$$6. \frac{3x^2+2x-5}{3x^2+11x+10} - \frac{x+1}{x-2} = \frac{2x+6}{x^2+5x+6} - \frac{8x-4}{x^2-4}$$

$$\frac{(\cancel{3x+5})(x-1)}{(\cancel{3x+5})(x+2)} - \frac{x+1}{x-2} = \frac{2(\cancel{x+3})}{(x+2)(\cancel{x+3})} - \frac{4(2x-1)}{(x-2)(x+2)}$$

$$\frac{(x-1)(x-2) - (x+1)(x+2)}{(x+2)(x-2)} = \frac{2(x-2) - 4(2x-1)}{(x+2)(x-2)}$$

$$\frac{x^2-3x+2 - x^2-3x-2}{(x+2)(x-2)} = \frac{2x-4 - 8x+4}{(x+2)(x-2)}$$

$$\frac{-6x}{(x+2)(x-2)} = \frac{-6x}{(x+2)(x-2)}$$