WORKBOOK

# Hare for Dinner?

## Learning Situation for MTH-4171

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Source: Bing.com

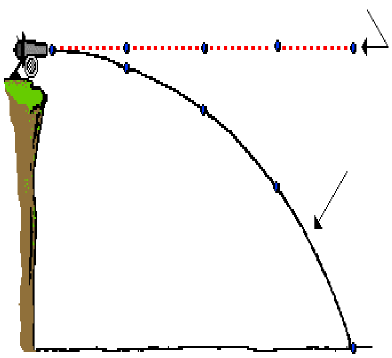
The purpose of this learning situation is to deduce from a table of values which algebraic model is being used. A detailed strategy is presented whereby the differences between two successive data items are calculated to determine whether the rule corresponds to an affine function or a quadratic function. A calculation strategy that uses a periodic function is also presented.

Table of Contents

## Scenario

Source: Bing.com

The study of movement began in Ancient Greece around the year 350 BC. At this time, theories were validated by logical reasoning and were handed down in the form of "great principles". However, observations on movement were described in such a way as to be open to interpretation. It was not until the 17th Century that these great principles and experimental results were clearly expressed in a universal language: mathematical models.



Source : Bing.com

During wars, spectacular advances have been made to models of movement. Initially theoretical, these models of movement were improved to take into account factors such as wind and air resistance. For instance, the aerospace industry has developed highly sophisticated models that precisely calculate an airplane's flight path.

In the Information Age, these algebraic models are used in video games, flight simulators and in the 3-dimensional reconstruction of accidents. Given that computers can do a vast number of calculations in a very short time, increasingly complex and realistic models can be put to use.

In this learning situation, you will construct algebraic models that describe a predator and its prey's movements in order to determine whether the prey will be caught. Likewise, you will figure out if the police can intercept a vehicle speeding down a highway. Further, in a different context to that of movement, you will use a calculation strategy using a model that changes over time.

## Unit 1 – Strategies for identifying a function from its table of values

A function can easily be recognized from its graphic representation. In this unit, you will learn strategies for identifying a function from its table of values.

### Identifying the affine function from its table of values

Here is the table of values for a function.  How can you identify which function it is?

| ***x*** | **f(*x*)** |
|---|---|
| 1 | 7 |
| 2 | 10 |
| 3 | 13 |
| 4 | 16 |
| 5 | 19 |

A smart way of finding out is to represent it graphically. However, there is another way of doing it and this is what is presented in this unit.

Look at the column of values for f(*x*). What regularity do you observe as you go down the columns from one line to the next?

| ***x*** | **f(*x*)** |
|---|---|
| 1 | 7 |
| 2 | 10 |
| 3 | 13 |
| 4 | 16 |
| 5 | 19 |

Difference

3

3

3

3

**We notice that there exists a constant difference between two successive values. When a table of values presents this regularity, this is known as an affine function.**

*<u>Note</u>: If you would like to further explore the content presented in this section, consult the Annex to this document.*

## Identifying the quadratic function from its table of values

Here is a table of values for a quadratic function. What regularity appears?

| *x* | f(*x*) |
|---|---|
| 0 | 7 |
| 1 | 12 |
| 2 | 23 |
| 3 | 40 |
| 4 | 63 |
| 5 | 92 |
| 6 | 127 |

The differences between the successive values of f($x$) are not constant. What can we observe about these differences?

| *x* | f(*x*) |
|---|---|
| 0 | 7 |
| 1 | 12 |
| 2 | 23 |
| 3 | 40 |
| 4 | 63 |
| 5 | 92 |
| 6 | 127 |

| Difference |
|---|
| 5 |
| 11 |
| 17 |
| 23 |
| 29 |
| 35 |

In fact, these differences are all spaced apart by the same number of units: we move 6 spaces to go from one number to the next. We say that the second differences between the values of f($x$) are constant.

| *x* | f(*x*) |
|---|---|
| 0 | 7 |
| 1 | 12 |
| 2 | 23 |
| 3 | 40 |
| 4 | 63 |
| 5 | 92 |
| 6 | 127 |

| First differences |
|---|
| 5 |
| 11 |
| 17 |
| 23 |
| 29 |
| 35 |

| Second differences |
|---|
| 6 |
| 6 |
| 6 |
| 6 |
| 6 |

**In the table of values for a quadratic function, the second differences between the successive values of f($x$) are constant.**

As with the affine function, it is essential to choose regularly spaced values of x for this regularity to appear.

Can this regularity be justified algebraically? Consider this particular example, where the values of x are spaced one unit apart. If the value is x, the next will be x+1 and the one after x+2. We can draw up the following table of values that represents the general behaviour of a quadratic function.

| ***x*** | **f(x)** |
|---|---|
| ***x*** | $f(x) = ax^2+bx+c$ |
| ***x***+1 | $f(x+1) = a(x+1)^2+b(x+1)+c$ |
| ***x***+2 | $f(x+2) = a(x+2)^2+b(x+2)+c$ |
| ***x***+3 | $f(x+3) = a(x+3)^2+b(x+3)+c$ |
| ***x***+4 | $f(x+4) = a(x+4)^2+b(x+4)+c$ |
| ***x***+5 | $f(x+5) = a(x+5)^2+b(x+5)+c$ |

The difference between f(*x*+1) and f(*x*) is:

$f(x+1) - f(x) = [a(x+1)^2 + b(x+1) + c] - [ax^{2\ 2} + bx + c]$

$f(x+1) - f(x) = [a(x^2 + 2x+1) + bx + b + c] - [ax^2 + bx + c]$

$f(x+1) - f(x) = ax^2 + 2ax + a + bx + b + c - ax^2 - bx - c$

$$f(x+1) - f(x) = 2ax + a + b$$

Find the difference between f(*x*+2) and f(*x*+1):

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the difference between f(*x*+3) and f(*x*+2):

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the difference between f($x$+4) and f($x$+3):

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and the difference between f($x$+5) and f($x$+4):

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Transpose these expressions into the table of values.

| *x* | f(*x*) |
|---|---|
| *x* | f($x$) = a$x^2$+b$x$+c |
| *x*+1 | f($x$+1) = a($x$+1)²+b($x$+1)+c |
| *x*+2 | f($x$+2) = a($x$+2)²+b($x$+2)+c |
| *x*+3 | f($x$+3) = a($x$+3)²+b($x$+3)+c |
| *x*+4 | f($x$+4) = a($x$+4)²+b($x$+4)+c |
| *x*+5 | f($x$+5) = a($x$+5)²+b($x$+5)+c |

| First differences |
|---|
| **2ax+a+** |
| **2ax+3a+** |
| **2ax+5a+** |
| **2ax+7a+** |
| **2ax+9a+** |

By calculating the second differences, we see that their expression is identical.

| *x* | f(*x*) |
|---|---|
| *x* | f($x$) = a$x^2$+b$x$+c |
| *x*+1 | f($x$+1) = a($x$+1)²+b($x$+1)+c |
| *x*+2 | f($x$+2) = a($x$+2)²+b($x$+2)+c |
| *x*+3 | f($x$+3) = a($x$+3)²+b($x$+3)+c |
| *x*+4 | f($x$+4) = a($x$+4)²+b($x$+4)+c |
| *x*+5 | f($x$+5) = a(v+5)²+b($x$+5)+c |

| First differences | Second differences |
|---|---|
| 2ax+a+ | **2a** |
| 2ax+3a+ | **2a** |
| 2ax+5a+b | **2a** |
| 2ax+7a+ | **2a** |
| 2ax+9a+b | |

Having verified the algebraic model of the function, we can deduce that this regularity is observable in **all** quadratic functions. For proof that this regularity is observable regardless of the difference in the values of $x$, consult the Annex.

Now, you are now ready to begin **Exercise 1**. You will be doing **Task 1** next.

**Exercise 1 Rain Gauges**

During a violent downpour, a weather technician is observing the data transmitted by two rain gauges situated in two different places in the town, one to the East and the other to the West. A rain gauge measures the height of the rainwater collected in a graduated cylinder topped by a funnel. Here is the data collected from the rain gauges over a period of 9 minutes.

**Readings from the two rain gauges from the start of the observation period**

**EAST Rain Gauge**

| Number of minutes | Height (mm) |
|---|---|
| 0 | 1,3 |
| 1 | 3,5 |
| 2 | 5,7 |
| 3 | 7,9 |
| 4 | 10,1 |
| 5 | 12,3 |
| 6 | 14,5 |
| 7 | 16,7 |
| 8 | 18,9 |
| 9 | 21,1 |

**WEST Rain Gauge**

| Number of minutes | Height (mm) |
|---|---|
| 0 | 0,4 |
| 1 | 2,2 |
| 2 | 5,4 |
| 3 | 10,0 |
| 4 | 16,0 |
| 5 | 23,4 |
| 6 | 21,8 |
| 7 | 20,2 |
| 8 | 18,6 |
| 9 | 17,0 |

To avoid flooding, the floodgates have to be opened when 10 mm more rain has fallen in the West than in the East. **From the start of the observation period, how much time has the technician got before he has to open the floodgates?**

To answer this question, you must determine the number of minutes that correspond to a difference of 10 mm between the two functions.

The question may be answered by using a graphical representation of the two functions. However, the algebraic method is shorter and more precise. First, find the type of function associated with each table of values by applying the techniques shown in Unit 1.

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You should have discovered out that the height of the rainwater in the EAST rain gauge behaves like an affine function. In the table of values for the WEST rain gauge, two groups of points are distinguishable. In the interval *[0, 5[* minutes, the height behaves like a quadratic function whereas in the interval *[5, 9]* minutes, it behaves like an affine function. Therefore, it is a function defined by parts.

Now, find the function rule using the following variables:

$y_{east}$ : height of the rainwater in the EAST rain gauge (mm)
$y_{west}$ : height of the rainwater in the WEST rain gauge (mm)
x: elapse of time during the observation period (min)

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The data from the EAST rain gauge are modeled by the function *yeast = 2,2x + 1,3*. Those of the WEST rain gauge behave like a function defined by parts:

$$\begin{cases} Yeast = 0{,}7x^2 + + {}^{2\ 22} + {}^{2} + \ +1{,}1x + 0{,}4 & \text{for } x \in [0, 5[ \\ Ywest = -\ 1{,}6x + 31{,}4 & \text{for } x \in [5, 9] \end{cases}$$

If you got the same results, congratulations! If not, check your rules by using another point on the table of values.

Now, subtract the two functions you have just obtained to discover which one gives the difference in height between the two rain gauges.

The function that gives the difference in height between the two rain gauges in the interval *[0,5[* is:

$Ywest - Yeast = (0,7x^2 + 1,1x + 0,4) - (2,2x + 1,3)$

$Ywest - Yeast = 0,7x^2 + 1,1x + 0,4 - 2,2x - 1,3$

$Ywest - Yeast = 0,7x^2 - 1,1x - 0,9$

In the interval [5,9], we have:

$Ywest - Yeast = (- 1,6x + 31,4) - (2,2x + 1,3)$

$Ywest - Yeast = - 1,6x + 31,4 - 2,2x - 1,3$

$Ywest - Yeast = - 3,8x - 30,1$

Determine the time when the technician opened the floodgates.

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The floodgates were opened 4.8 minutes after the technician started his observations. If you did not get this result, take the time to go over your answer to find out where you made an error.

It can be noted that the rule for the WEST rain gauge in the interval *[5,9]* did not need to be used to answer this question. Thus, it was unnecessary to model this part of the data. An analysis of the table of values shows the difference in height between the WEST and EAST rain gauges was 11.1 mm after 5 minutes. If we had made this observation before we modelled the functions, we would have known immediately that the difference of 10 mm would have been attained in the interval *[0,5[* . Therefore, we only needed to look for the rule in this interval.

**Now that you have understood this exercise, let's get started with Task 1.**

# Scenario for Activities 1 and 2

**Habits of the hare and the lynx**

The American Hare is a very fast animal. When he senses danger, he leaps and runs in zigzags to confuse his predator. These leaps can be more than 3 metres high and his top speed can reach up to 60 km/h. He achieves this speed in record time but is not able to sustain it for too long.

Source: Bigstockphoto.fr

The doe, the female hare, can have two or three litters a year. She gives birth to 3 or 4 young in each litter. Thus, it is a very prolific animal. A hare lives, on average, 4 to 5 years. The lynx is his main predator.

The lynx is a formidable predator. He has very acute hearing and can detect even the slightest of sounds in his environment. He also has very sharp eyes that help him see his prey, even at night. He is very fast: obtaining speeds up to 60 km/h. 80% of a lynx's diet is composed of hares. He can eat up to 200 of them a year.

Source: Bigstockphoto.fr

The hare population changes with each cycle of 9 or 10 years. When the population of hares increases, the number of its predators increases likewise. Eventually, their habitat can no longer provide all the food that the hares need. Weakened, they cannot escape their predators so easily. This lack of food also affects the fertility of the females and the survival of their young. The population of hares then starts to decrease. The number of predators also decreases because their source of food is diminished. This is a natural cycle.

External factors can disturb this cycle. Over hunting is one of these. It is, therefore, primordial that the wildlife conservation officers monitor the number of hares killed during hunting season and intervene should the threat of extinction occur.

## Task 1 – Hare for dinner?

A hare is exploring a wooded area in search of food when he hears a cracking sound. He spies a lynx who tries to pounce on him. The hare runs away to escape his predator.

The following data predicts the distance run over time for each animal and are based on the movements normally observed in hunting situations.

**Distance run by the lynx over the time**

| Time (s) | Distance run (m) |
|---|---|
| 0 | 0 |
| 0,5 | 8 |
| 1 | 16 |
| 1,5 | 24 |
| 2 | 32 |
| 2,5 | 40 |
| 3 | 48 |
| 3,5 | 56 |
| 4 | 64 |
| 4,5 | 72 |
| 5 | 80 |

**Distance run by the hare over the time**

| Time (s) | Distance run (m) |
|---|---|
| 0 | 8 |
| 0,5 | 12 |
| 1 | 17 |
| 1,5 | 23 |
| 2 | 30 |
| 2,5 | 38 |
| 3 | 46 |
| 3,5 | 54 |
| 4 | 62 |
| 4,5 | 70 |
| 5 | 78 |

The instant the hare begins to flee, the lynx is some distance behind him. **If the animals move as described, will the lynx have hare for dinner?**

**To be able to catch his prey, how close must the lynx get without being spotted by the hare?**

## Reflection

What does it tell you when several regularities can be observed in the same table of values?

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How have you, or how would you, check the results obtained in this task?

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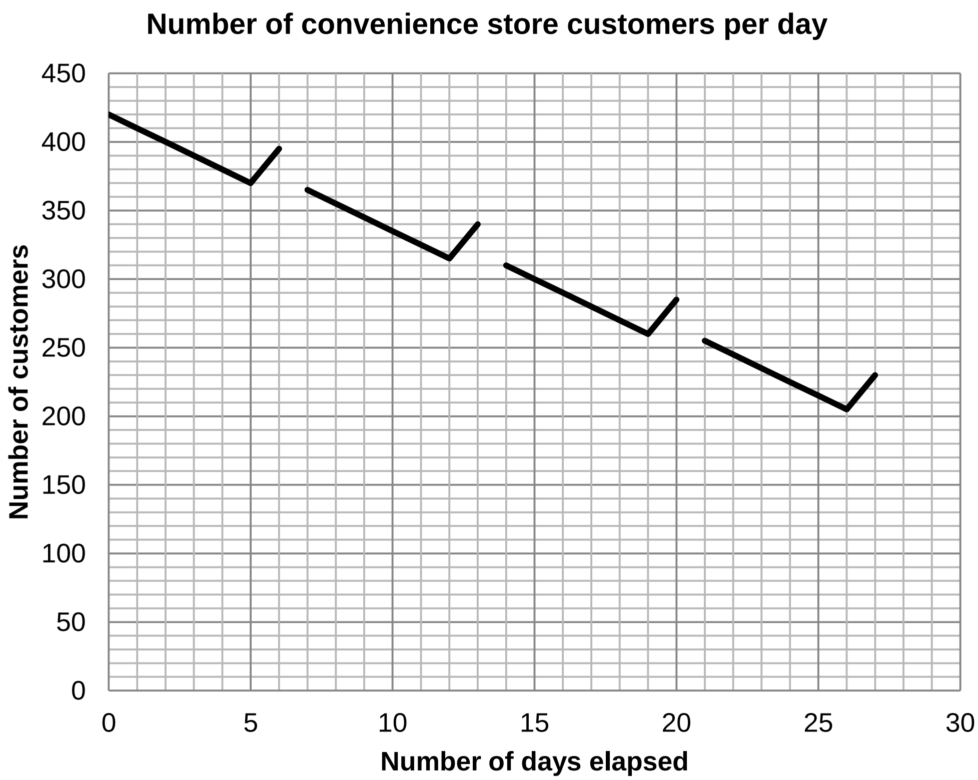
## Unit 2 – A periodic function

In this unit, we employ a calculation strategy that uses a periodic function. You will find out about this strategy as you solve the next problem.

**A convenience store with few customers**

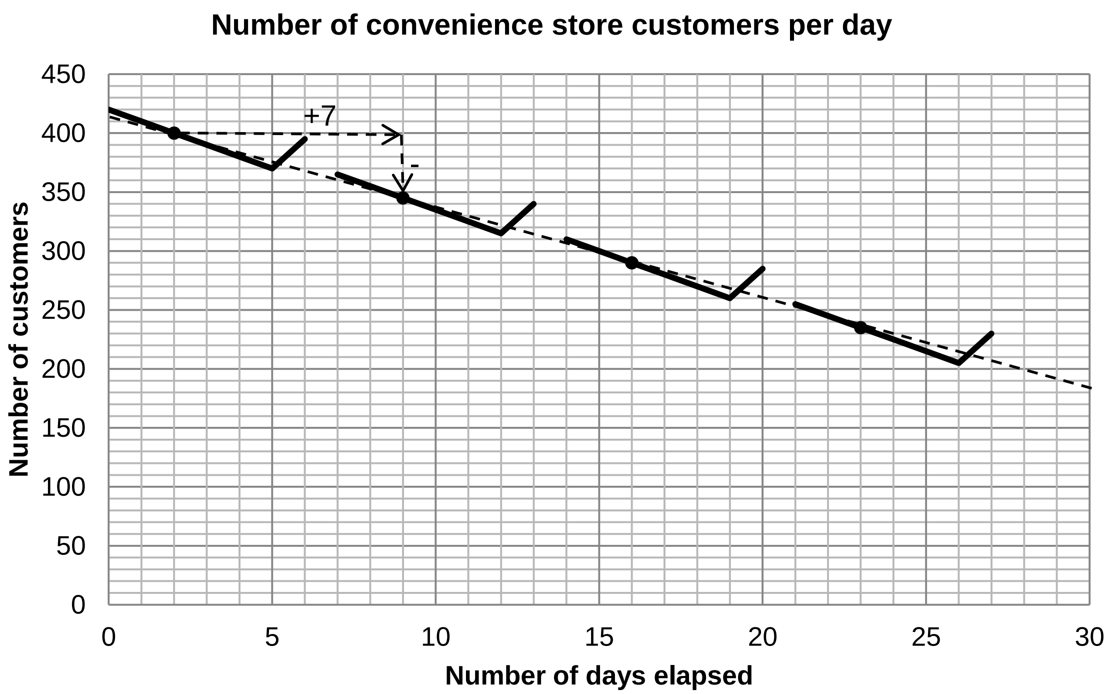
The number of customers of a convenience store decreases from week to week because a competitor has opened several blocks away. The convenience store owner keeps a record of the number of customers he serves each day. Here is the data from the last 4 weeks.




**His business closes on the 51st day. If the trend observed had continued, how many customers would he have received on the day he closed?**

An analysis of the graphic representation allows us to see that the function as defined by parts situated in the interval [0, 6] is repeated every 7 units on the x axis following a vertical translation of – 55 units.

The points that are in the position equivalent to the 51st day are obtained by moving back a complete number of 7-day cycles. These points are those where the x-coordinate is equal to: 51, 44, 37, 30, 23, 16, 9 and 2. Draw a line that passes through these points.




This line passes through the point (2, 400) whose coordinates can be read on the graph. With a horizontal translation of 7 units and a vertical translation of - 55 units at this point, we obtain the coordinates of the point that is in an equivalent position in the following interval:

(2+7, 400-55) = (9, 345).

Using the coordinates of these two points, the line's rate of change can be determined:

$$a = \frac{345 - 400}{9 - 2} = \frac{-55}{7} \approx -7.86$$

$$\approx a = \frac{345 - 400}{9 - 2} = \frac{-55}{7} \approx -7.86$$

$$a = \frac{345 - 400}{9 - 2} = \frac{-55}{7} \approx -7.86$$

The rule is expressed as:

$y = ax + b$

$y = - 7,86x = b$

The value of *b* is obtained by substituting the coordinates of one of the points in the equation. With the point (2, 400), we obtain:

$$400 = 2 - 7,86 + b$$

$$400 = -15,72 + b$$

$$415,72 = b$$

Thus, the equation for the line is:

$y = - 7,86x + 415,72$

All that is left to do is to determine the value of the function at $x$=51:

$y = -7,86x51 + 415,72$

$y \approx 14,76$

**The convenience store owner serves 15 customers on his final day of business.**

You will have the opportunity to use this strategy in the next task.

You can now start **Task 2**.

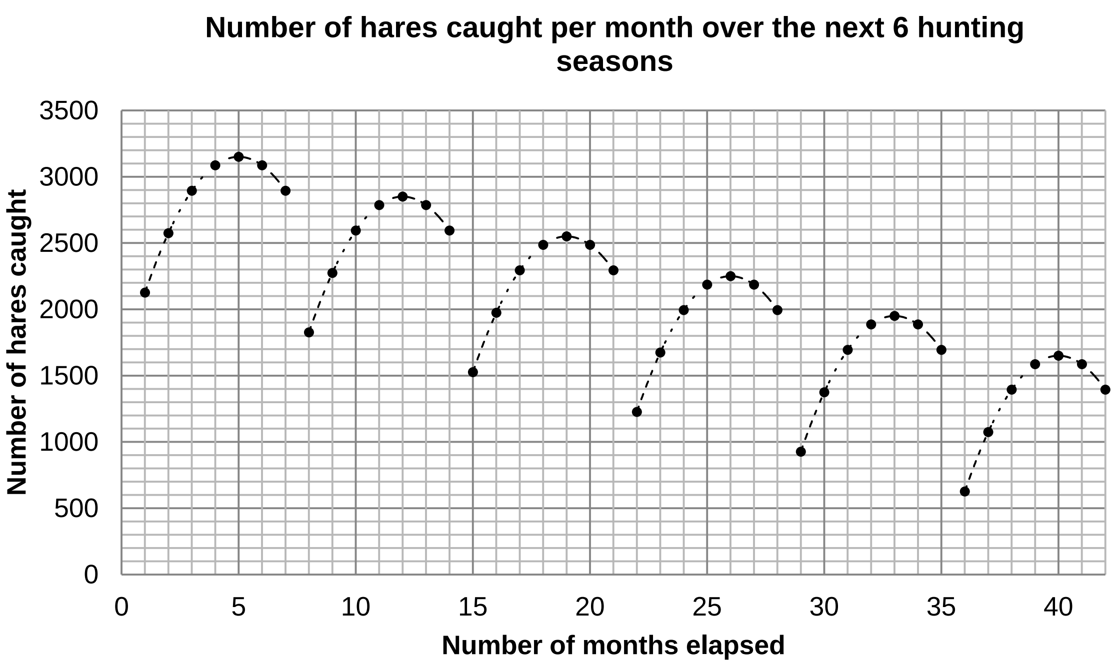
## Task 2 – Advance planning for a hunting tournament

You have already discovered the rule for a quadratic function from its graphic representation. Now, let's see how you will get along with this function whose parabola changes over time.

Hare hunting is a sport that is gaining in popularity on a Northern Québec wildlife reserve. Ecologists have noted that a smaller number of hares have been caught, despite an increase in the number of hunters. From this, they have deduced that the hare population is in a period of serious decline.

The ecologists asked if they could consult the data collected by the wildlife conservation officers. From an analysis of this data, they have deduced that the maximum number of hares caught fell by 300 each season. To predict how many hares would be caught over the next six hunting seasons, this is the model they constructed.

Note: A hunting season runs from September 1 to March 31.




An important hunting tournament is being organized for December 2020. **How many hares can they hope to catch at this event if there has already been a maximum of 3,150 caught during the 2014-2015 season and it is predicted that 1,526 will be caught at the beginning of the 2016-2017?**

## Reflection

Make a 4-step summary of your solution.

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How do you convert the month of a year to a value on the x axis?

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## Task 3 – A police chase on the highway

The police are looking out for speeding vehicles and are hidden on the edge of the highway. They notice a speeder and take off at full throttle to intercept him. The speeder does not slow down. He continues to drive at top speed hoping to get away from the police who are far behind him.

The tables of values below describe the movements of the two vehicles.

**Distance travelled over time**
**Speeder**

| Time (s) | Distance (m) |
|---|---|
| 0 | 250 |
| 1,5 | 332 |
| 3 | 414 |
| 4,5 | 496 |
| 6 | 578 |
| 7,5 | 660 |
| 9 | 742 |
| 10,5 | 825 |
| 12 | 907 |
| 13,5 | 989 |
| 15 | 1 071 |
| 16,5 | 1 153 |
| 18 | 1 235 |
| 19,5 | 1 317 |
| 21 | 1 399 |

**Distance travelled over time**
**Police**

| Time (s) | Distance (m) |
|---|---|
| 0 | 0 |
| 1,5 | 5,2 |
| 3 | 20,9 |
| 4,5 | 47,0 |
| 6 | 83,5 |
| 7,5 | 130,5 |
| 9 | 187,9 |
| 10,5 | 255,7 |
| 12 | 334,1 |
| 13,5 | 422,8 |
| 15 | 516,6 |
| 16,5 | 610,3 |
| 18 | 704,1 |
| 19,5 | 797,8 |
| 21 | 891,6 |

**Is the speeder far enough ahead of the police to be able to get away from them?**

**How far ahead must the speeder be from the police to get away?**

**This kind of behaviour can lead to a driving licence being suspended for a year for a first offence.**

## Reflection

Compare the strategies used in Tasks 1 and 3.

Imagine another problem that could be solved using the same strategy.

What resources did you consult during this learning situation? Which ones would you use again?

# A reminder

## Unit 1 – Strategies for algebraic modelling using a table of values

The **regularities** to look for are:

- **Affine**: a constant difference between successive values of f($x$)
- **Quadratic**: second constant differences between successive values of f($x$)

✓ Choose the **regularly spaced** values of $x$ to find the regularity.

Suggestion 1: Before starting your calculations examine the **data.** You will make some useful deductions.

Suggestion 2: It is always wise to **check** your result by solving the problem using a different method.

## Unit 2 – A periodic function

- Connect equivalent points
- Find the coordinates of two points by taking advantage of the regularity in the translations
- Find the rule of the line
- Evaluate f($x$)

## Annex

### Digging deeper into Unit 1

### Identifying an affine function from its table of values (continuation)

Using the regularity observed on Page 3, complete the following table of values.

| *x* | f(*x*) |
|---|---|
| 1 | 7 |
| 2 | 10 |
| 3 | 13 |
| 4 | 16 |
| 5 | 19 |
| **6** | |
| **7** | |
| **8** | |
| **9** | |

Now, choose several points from this table of values.

| *x* | f(*x*) | Difference |
|---|---|---|
| 1 | 7 | |
| 3 | 13 | 6 |
| 4 | 16 | 3 |
| 7 | 25 | 9 |
| 9 | 31 | 6 |

Even though not all values are there, the function is still the same. In fact, if we put these points on a Cartesian Plane, they would be in a line. However, the regularity observed above would no longer appear… Why do you think that is?

In fact, the first time we calculated the difference between the values of f(*x*), the values of *x* regularly increased. This is not true for this table of values. Thus, when we are looking for the regularity of a function in its table of values, it is very important to ensure that the values of f(*x*) are related to the **regularly spaced values of *x*.**

Let’s make a final observation. Now, evaluate the differences between the values of *x*.

| Difference | ***x*** | **f(*x*)** | Difference |
|---|---|---|---|
| 2 | 1 | 7 | 6 |
| 1 | 3 | 13 | 3 |
| 3 | 4 | 16 | 9 |
| 2 | 7 | 25 | 6 |
| | 9 | 31 | |

If we evaluate the relationship between the differences in the values of f(x) and the differences in the related values of x, we see that its value is constant:

$$\frac{Difference\ of\ f(x)}{Difference\ of\ x} = \frac{6}{2} = \frac{3}{1} = \frac{9}{3} = \frac{6}{2} = 3$$

This relationship corresponds to the rate of change of the function. In effect:

$$rate\ of\ change = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Yet, as learned in the MTH-3051 course, an affine function has a constant rate of change. This characteristic appears in its table of values as a constant difference between the values of f(*x*) associated with regularly spaced values of *x*.

## Modelling the affine function

Find the rule that corresponds to the table of values presented to you in this section.

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Verification

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## Identifying the quadratic function from its table of values (continuation)

To expand on what was covered on pages 5 and 6, suppose the values of x were spaced by any number of units, such as h. The table of values would look like this.

| $x$ | f($x$) |
|---|---|
| $x$ | $f(x) = ax^2+bx+c$ |
| $x+h$ | $f(x+h) = a(x+h)^2+b(x+h)+c$ |
| $x+2h$ | $f(x+2h) = a(x+2h)^2 +b(x+2h)+c$ |
| $x+3h$ | $f(x+3h) = a(x+3h)^2 +b(x+3h)+c$ |
| $x+4h$ | $f(x+4h) = a(x+4h)^2 +b(x+4h)+c$ |
| $x+5h$ | $f(x+5h) = a(x+5h)^2 +b(x+5h)+c$ |

As before, calculate the first and second differences.

The difference between f($x$+h) and f($x$) is:

$$f(x + h) - f(x) = [a(x + h)^2 + b(x + h) + c] - [ax^2 + bx + c]$$

$$f(x + h) - f(x) = [a(x^2 + 2hx +h^2) + bx + bh + c] - [ax^2 + bx + c]$$

$$f(x + h) - f(x) = ax^2 + 2ahx +ah^2 +bx + bh + c - ax^2- bx - c]$$

$$f(x + h) - f(x) = 2ahx + ah^2 + bh$$

Find the difference between f($x$+2h) and f($x$+h):

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the difference between f($x$+3h) and f($x$+2h):

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the difference between f($x$+4h) and f($x$+3h):

and the difference between f($x$+5h) and f($x$+4h):

Transpose these values to the table below and calculate the second differences.

| $x$ | f(x) |
|---|---|
| $x$ | f($x$) = a$x^2$ |
| $x$+h | f($x$+h) = a$(x+h)^2$ |
| $x$+2h | f($x$+2h) = a$(x+2h)^2$ |
| $x$+3h | f($x$+3h) = a$(x+3h)^2$ |
| $x$+4h | f($x$+4h) = a$(x+4h)^2$ |
| $x$+5h | f($x$+5h) = a$(x+5h)^2$ |

First differences

- $2ahx+ah^2+bh$
- $2ahx+3ah^2+bh$
- $2ahx+5ah^2+bh$
- $2ahx+7ah^2+bh$
- $2ahx+9ah^2+bh$

Second differences

- $2ah^2$
- $2ah^2$
- $2ah^2$
- $2ah^2$

As h can be of **any** value, our conclusion is valid **no matter what the difference** is between the values of $x$. Therefore, we can confirm that the values of the second differences of f($x$) in the table of values of a quadratic function is always constant.

## Modelling the quadratic function

Find the function rule for the table of values presented in this section.

Verification