

2017-12-28

MTH-4161

**Algebraic and Graphical Modelling in an Applied
Context 1**



Bullseye!

Teacher's Guide

Created by François Guay-Fleurent

Translated by Marie-Hélène Lebeault

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Author

François Guay-Fleurent, teacher, CS de la Riveraine, June 2016.

guayfleurf@ecole.csriverraine.qc.ca

I would love to hear your comments, questions, modification ideas, etc.

Educational Aims

- Work on processes and strategies for problem-solving in mathematics, with a touch of scientific research.
- Discover and manipulate, in a real situation with concrete experimentation, a second-degree polynomial function.
- Even before cracking open a workbook, introduce the student to the Technical and Scientific Option. This first contact, I hope, will be motivating.
- Process a situation that requires a representation using an algebraic or graphical model expressing a dependency between quantities, in an applied perspective.
- Continue the development of the student's three mathematical competencies by considering three aspects: Acting in context, Mobilization of Resources and Reflection.

Target Audience

Course: MTH-4161-2 Algebraic and Graphical Modelling in an Applied Context 1

When to Administer: As the first learning situation, even before starting the workbook.

Learning Situation Duration

About 4 to 8 hours, but ideally the shortest time possible. Working with a classmate or the teacher will reduce the time required to complete the learning situation.

Type of Task

Learning Situation (LS) or Problem Situation (PS)

Learning Situation Description

After having identified the problem or challenge, activated prior knowledge, and established an action plan, the student will go to the laboratory to collect data.

Then, he or she will analyze the data and create the necessary algebraic or graphical models to meet the challenge.

Finally, in collaboration with the teacher, he or she will carry out the challenge, which is to hit the target using mathematics.

Important information

This LS was inspired by Discovery Learning (see Internet references below), in that it brings students to construct their own representation of a problem-situation using an algebraic or graphical model and to discover the second-degree polynomial function and a few of its properties. Students, ideally, will perceive the use of mathematical modelling, and should, in subsequent problem-situations, better understand « why we're doing this ».

To reduce the student's workload, teachers are invited to simplify the LS as needed. One way would be to do whole sections verbally with students such as the action plan or the reflection. This way, the teacher can guide students while providing opportunities for reflection at the right times, which is sometimes hard to do with a paper-based assessment. Another way to shorten the LS is to prepare the material in advance for data collection. If you have one, it's best to work with the lab technician. If you don't, you could use the same assembly with every student, prepared initially by a student or the teacher.

Also, it is highly recommended to have students work in pairs, though the LS can be done independently.

LS Context

Studying projectile firing is not a new thing. Whether it's arrows, catapults or golf, Man has always wanted to know where his projectiles will land.

And that was one of the objectives of the Munitions Experimental Test Centre at Nicolet. For 50 years, the centre tested ammunition by firing shells directly in St-Pierre lake (St-Lawrence River).

How can we predict the trajectory of the projectiles and therefore predict where they will hit?

Expected Final Production

This problem-situation requires you to go to the lab to experience a projectile shot subjected to a gravitational force to meet your teacher's challenge: hit the target on the first try using mathematics! While you're at it, you'll discover new mathematical functions that are quite useful in the study of ballistics.

Broad Areas of Learning and Focuses of Development

Environmental Awareness and Consumer Rights and Responsibilities.

Knowledge of the environment

Understanding of certain characteristics of the human environment

At the end of this LS, the student will have a better understanding of the trajectory of a projectile and the effects of gravitational forces.

Cross-Curricular Competencies

Intellectual competencies: Solves problems.

Methodological competencies: Adopts effective work methods.

Personal and social competencies: Cooperates with others.

Communication-related competency: Communicates appropriately.

Subject-Specific Competencies

Competency 1: Uses strategies to solve situational problems

Competency 2: Uses mathematical reasoning

Competency 3: Communicates by using mathematical language

The LS follows the guidelines of the Program (p. 19) by fostering the development of each of these competencies by considering three aspects: acting in context, the mobilization of resources and reflection.

Integrative Processes

Using an algebraic or graphical model to represent a situation.

Interpolating or extrapolating from an algebraic or graphical model.

Prescribed Knowledge

Mathematical Knowledge in the LS

Manipulating numerical and algebraic expressions

- Solving equations and inequalities in one variable: second-degree

Function and inverse

- Experimenting with real functions as well as observing, interpreting, describing and representing them
- Describing and interpreting the properties of real functions

Describing and interpreting the properties of real functions

- Domain and codomain
- Increasing and decreasing
- Extrema
- Sign
- X- and Y- intercepts

Material Resources

Material for manipulation

The launch ramp can be constructed quickly with a coroplast pasted on a box. The height of the box will determine the speed of the projectile and the opening of the parabola. Choose according to the space you have on hand. Make a notch in the ramp so you can fire the projectile from the same spot every time. The projectile can be a marble, a small hard ball, an old computer mouse ball or any other perfectly round object that is firm. The target holder can be a board supported by brackets or can even be the pegboard used in the examination for the course TSC-4063. The target holder should be marked or graduated to facilitate the identification of projectile impacts. You could also add a layer of soft material to the target holder, which would leave a mark upon impact. Be sure that the target holder is always kept vertical. You could also choose to invert the parameters by finding h and measuring d .

However, once the projectile has a good speed, it's easier if the target is vertical. A computer with spreadsheet software speeds up the creation of the table of values and will be useful to produce the scatter plot graph.

Any other material additions are at the discretion of the teacher. For example, you could use a projectile launcher instead of a launch ramp. Also, the target could be a laundry detergent cap attached to a universal holder with a clamping nut. That way, you could adjust the target until you have three convincing trials. It would make the LS easier, though longer, to do it independently. Finally, you could use carbon paper to mark the impact of the projectiles. However, all these additions may require extra time from the teacher and the student.

***See assembly example pictures in the Appendix.**

Encyclopedic Resources

Workbook - Teacher's choice. This material is not essential for the smooth running of the LS.

Internet Resources (in French)

Discovery Learning (Apprentissage par découverte)

Édu Tech Wiki (Université de Genève):

http://edutechwiki.unige.ch/fr/Apprentissage_par_d%C3%A9couverte

UQAC: <http://www.uqac.ca/deptdse/3psy206/facapp/decouv2.html>

UQTR. Though dated, the rest of the website is also interesting:

http://www2.uqtr.ca/hee/site_1/index.php?no_fiche=2604

Tutorials prepared by the author found in the Appendices (in French)

Learning to contextualize data in Excel (4 min):

<https://youtu.be/QN4hSBgYhiA>

Forcing a linear trend line through the origin in Excel (1 min):

<https://youtu.be/nJRMRVlhVhI>

Desmos

To quickly graph functions on a computer or tablet: <https://www.desmos.com/calculator>

<https://itunes.apple.com/us/app/desmos-graphing-calculator/id653517540?mt=8>

Create graphs and add a trend line

Written Tutorials:

Office Excel Tutorial - Present your data in a scatter chart or a line chart:

<https://support.office.com/en-us/article/Present-your-data-in-a-scatter-chart-or-a-line-chart-4570A80F-599A-4D6B-A155-104A9018B86E?omkt=en-CA&ui=en-US&rs=en-CA&ad=CA>

Office Excel Tutorial to add, change, or delete a trend line and to show an equation and the correlation coefficient:

<https://support.office.com/en-us/article/Add-a-trend-or-moving-average-line-to-a-chart-fa59f86c-5852-4b68-a6d4-901a745842ad?omkt=en-CA&ui=en-US&rs=en-CA&ad=CA>

Office Excel Tutorial - Choosing the best trend line for your data:

<https://support.office.com/fr-fr/article/Choisir-la-meilleure-courbe-de-tendance-pour-vos-donn%C3%A9es-1bb3c9e7-0280-45b5-9ab0-d0c93161daa8>

Video Tutorials:

Office Excel Tutorial - Making a scatter Plot Graph:

<https://www.youtube.com/watch?v=DlyU6V1VleE>

Office Excel 2010 Tutorial - How to create scatter plot with trendline equation and correlation of determination: <https://www.youtube.com/watch?v=cxoYa0xSng8>

Office Excel Tutorial - How to add trendline, equation in excel 2016:

<https://www.youtube.com/watch?v=7KH5q4fLgtE>

Office Excel Tutorial - How to add equation to trendline in Excel

<https://www.youtube.com/watch?v=3hLmYq0i0Ww>

Office Excel Tutorial - Getting a Trendline, Equation, and R-squared value:

<https://www.youtube.com/watch?v=Ek7BhjJGcEw>

Overview

Preparation

- The adult reads the context;
- Definition of the problem;
- Activate prior knowledge;
- Student establishes a plan of action;
- The teacher checks this part of the student's work and makes a first assessment. Immediate feedback should be given to the student.

Production

- Data Collection at the lab;
- Data analysis;
- Create a model to meet the challenge.

Integration

- Carrying out the challenge;
- Reflection;
- Exercises in appendices if deemed interesting by the teacher.

Reinvesting Knowledge

The processes and strategies used to solve the situational problem will be reinvested in subsequent problem-situations. Also, using a spreadsheet, if it was a new experience for the student, could be reinvested in other algebraic or graphical modelling activities.

Assessment

The teacher should assess the adult's booklet and make a judgement on the development of his / her skills. We strongly suggest using the MEES Evaluation Rubric. That way the student knows the evaluation criteria for the upcoming ministry examination. The answer key will help teachers to make their judgement and they could also design an observation checklist, as needed. Bear in mind that, at the beginning of a course, assessments to support learning will target general points such as processes instead of specifics such as mathematical concepts.

Notes

If you don't have enough room to carry out the data collection, you can use the data provided in the Answer Key or in the Student Data Spreadsheet. However, the LS will likely not be as interesting for the student. I would still being relevant though.

You can use OpenOffice Calc, Excel, Libre office Calc, Géogébra, Desmos, among others to produce a table of values, a scatter plot, and graph trend lines and correlation coefficients. I used Excel.

Experimental error is completely normal in this type of experiment. That's why it's important to do at least three trials and calculate the average of the measurements. Also, the lower the impact on the target holder, the harder it will be to measure. It might be more efficient to do more than three trials to have more accurate data. The important thing in this LS is not so much the accuracy of the values, but the modelling and reflection around it. The point is not to get a perfect curve.

You have two options when posing the challenge to the student at the end of the LS.

- 1- Once data collection and modelling are completed, you can choose a distance (d) in their range of data for which they have no data. In this instance, they will have to interpolate to identify the point of impact.
- 2- Right at the start, you could tell them not to go further than a certain height (h) or a certain distance (d). That way you could choose a distance (d) out of the domain. Then, the student will need to extrapolate to identify the point of impact. You could also have them predict where the projectile will land on the ground? That way, they can extrapolate graphically with Excel as well as algebraically. You could also pose one, two, or three different challenges. It's up to you to assess the student's interest for it.

Appendices

For examples of possible answers, look for the Answer Key on the Alexandrie FGA website.

There are a few exercises in the Appendices.

Comments on the different parts of the Adult Booklet.

1. REPRESENTING THE PROBLEM

The student must identify the problem, represent it, and activate prior knowledge.

Question you could ask the student: Did you validate your comprehension of the problem with another person?

2. PLANNING THE PROBLEM-SOLVING

This section is where the student collects data and establishes a global plan of action to solve the LS. They design a table of values, and adapt the assembly plan to their liking. You can work with the student for this section to save time, if possible. Also, the student will have a clearer understanding on how to proceed. However, you may want the student to design the whole assembly, as you would in the course Science and Technology, it's up to you. You should then remove the diagram from the Adult Booklet and ask them to produce it.

3. ACTIVATING THE PLAN OF ACTION

The student goes to the laboratory to collect data. At the laboratory or once back in class, they update their plan of action and analyze data with the spreadsheet thus producing an algebraic and graphical representation of the LS and meet the challenge.

While analyzing data, they may need to consult the teacher, the technician or the Internet resources provided in this guide to learn how to design a clear graphical representation with a trend line, an equation, and a correlation coefficient (R^2).

4. REFLECTING ON STRATEGY

It is possible that the software, to follow the trend, shifts the peak of the parabola toward the first quadrant instead of leaving it centred at the origin. If the student didn't notice or didn't adjust it, you can provide them with the document Tips for using the spreadsheet. The small document will help the student simplify their function and the corresponding equation.

CARRYING OUT THE CHALLENGE

For notes on the challenge, see Notes on pages 8 and 9. I personally think this is the best part of the LS where the student is in action and uses mathematical tools combined with logic to predict where the projectile will hit the target. In real life, it's the opposite. The shooter or the golfer know where the target is and must make the necessary adjustments and calculations to hit the target. This realistic situation would have been interesting to do, but we'll leave that challenge for the physics class. ☺

LET'S REFLECT

I believe this section should be done with the teacher to save time and to add weight to the reflection, otherwise it will likely be done in haste. Also, the teacher could ask additional questions according to the student's responses.

Finally, a short video, unfortunately both informative and enticing, introduces artillery jobs in the army. It shows how the army uses technical equipment and mathematics in their tasks.

Conclusion

Thank you for sending me your comments, your pictures, your improvements to the documents, your ideas, and your questions. I will gladly respond to all of them.

I would also like to thank Isabelle Trépanier (CSDM), Yves Khouzam (CSMB), Paul Trépanier, and Luc Favreau (ÉTS) who came up with the idea to model the firing of a projectile.

François Guay-Fleurent

Teacher

Pedagogical Consultant for Récit FGA – Region 17

819 293-5821 #6730

guayfleurf@ecole.csriveraine.qc.ca

Appendix 1 – Simple example of a possible assembly



The launch ramp. Ideally, it should be higher and longer. It should also be taped or glued. The cardboard at the base acts as a transition between the ramp and the table to avoid rebounding.

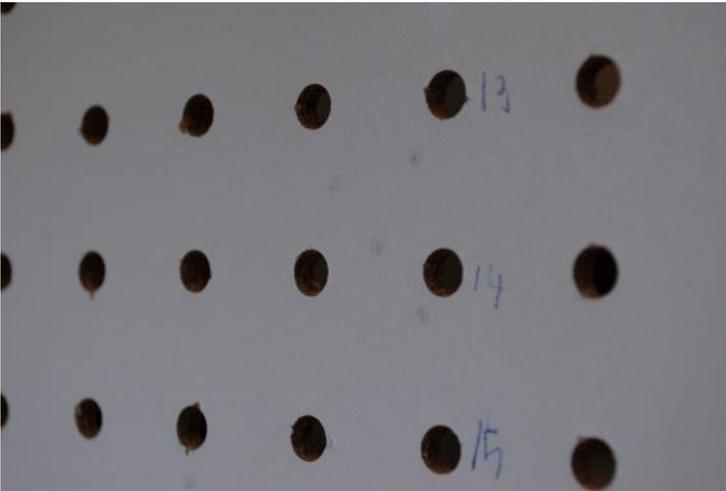
A hole in the wood lets you place the marble in the same starting point.



And it's off!



Marbles are too bouncy. The grey mouse ball is too heavy, but better. The black ball from an old mouse was perfect. It had a good mass and left little marks on the white pegboard. It was fantastic. And, it barely bounced.



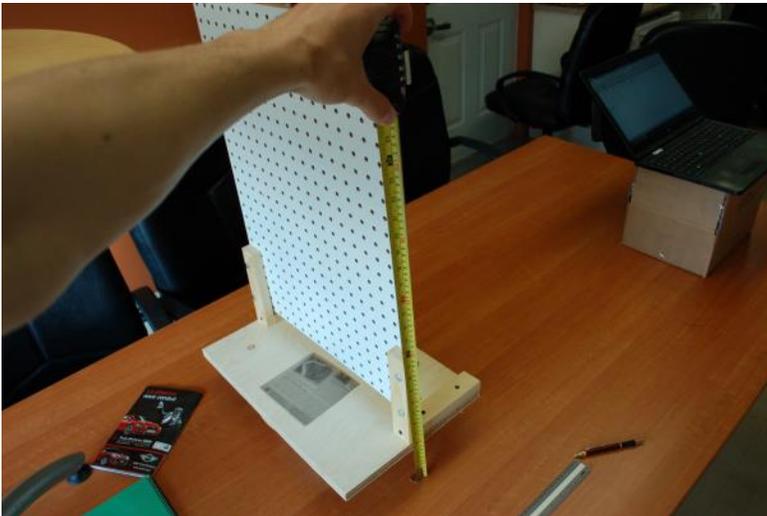
Little black marks and numbering of the holes.



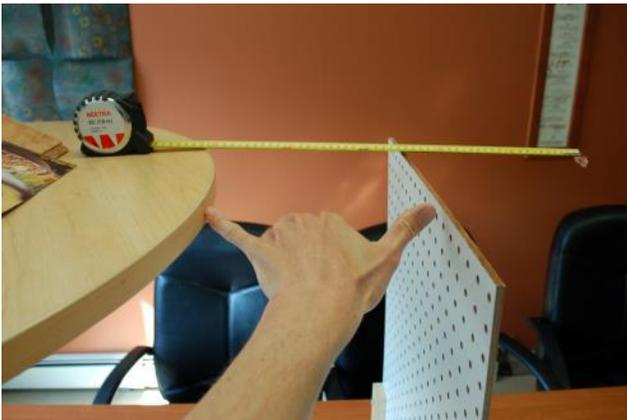
Overview of the data collection assembly. It took about 10 minutes to make the assembly and 30 minutes to collect data. I did it alone. I preferred working on a table to avoid having to bend over a lot. However, the students can use the lab table and the floor. It's up to you.



Pointing the origin. That's where the projectile left the table and started free falling. $F = m * g$ is the only force applied to the projectile, if you don't consider air resistance.



It was easier to measure from the ground. Then you can easily convert the data to contextualize them. See Appendix 2 for that.



This is the distance between the start of the free fall and the target (d). It's the independent variable in the experiment.



The computer was on hand to input data in the spreadsheet.

Appendix 2 – Tips for contextualizing with a spreadsheet

Look at the following video (in French) to learn how to contextualize your data with a spreadsheet.

<https://youtu.be/QN4hSBgYhiA>

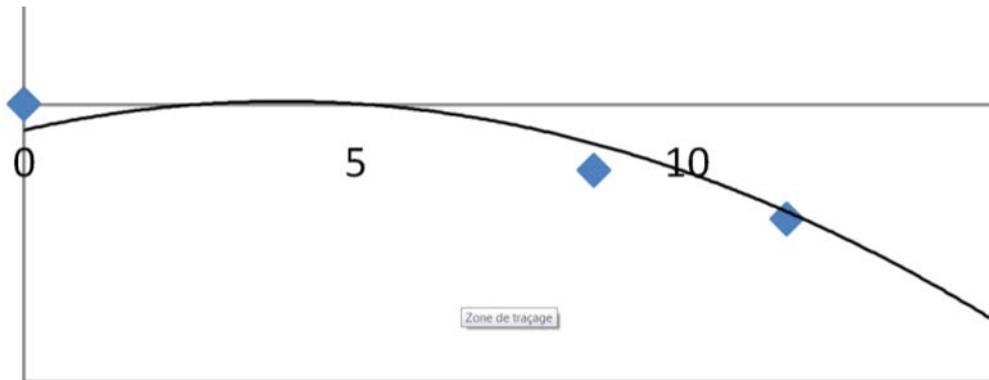


In this video, we show you how to quickly contextualize data using a spreadsheet, that is to change the measurements between to ground and the impact point to those between the x-axis and the impact point.

You also learn how to quickly calculate the average of the three results.

Appendix 3 – Tips with a spreadsheet to obtain the rule in the form $y = ax^2$

Enlarging the graph



When you enlarge the graph, you notice that the peak is not where it should be at (0,0) because it's a trend line. So the start point isn't at (0,0) either. Furthermore, it's unlikely that the marble goes up before it goes down in our experiment.

We will force Excel to use (0,0) as the origin. Look at the video (in French) to learn how to proceed:

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That's how you get the « new » rule:

$$y = -0.0334x^2 + 0.2256x$$

This rule is simpler than the first one ($y = -0.0342x^2 + 0.2653x - 0.4613$). But that is cheating a little.

If we continue to cheat to obtain the form $y = ax^2$, we get:

$$y = -0.0334x^2 + 0.2256x$$

$$y = -0.0334x(x - 6.75)$$

If we state that the square root of 6.75 is in fact 0, we get:

$$y = -0.0334x(x - 0.00)$$

$$y = -0.0334x^2$$

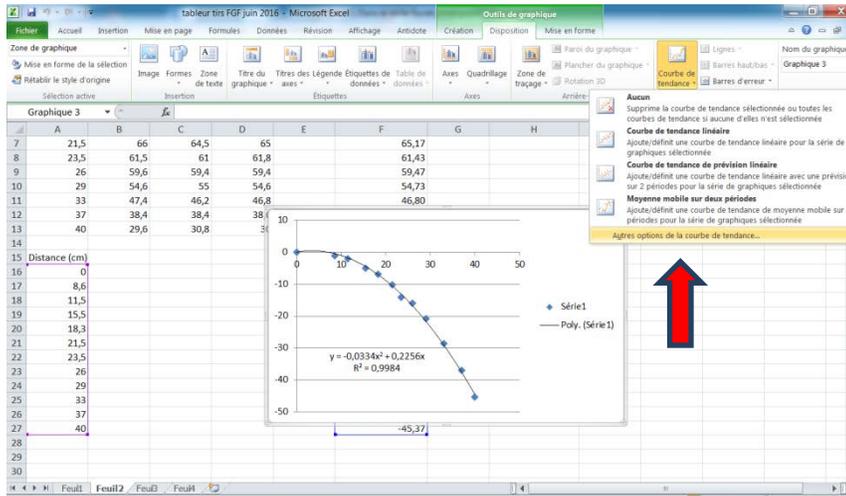
This simplified and slightly shifted form of the function can be graphed using a graphing calculator or with Desmos so you can observe it. Furthermore, I believe if there was more force to the firing (greater initial speed of the projectile), this simplification would have a lesser impact on the pace of the parabola. In this case, we can't use the simplification to meet the challenge because the model is no longer accurate enough.

Using the other two forms, less simplified but closer to reality, are recommended to meet the challenge. It will be more accurate. However, for the exercises at the end of the LA, the simplified form is best to learn about the properties of the polynomial function.

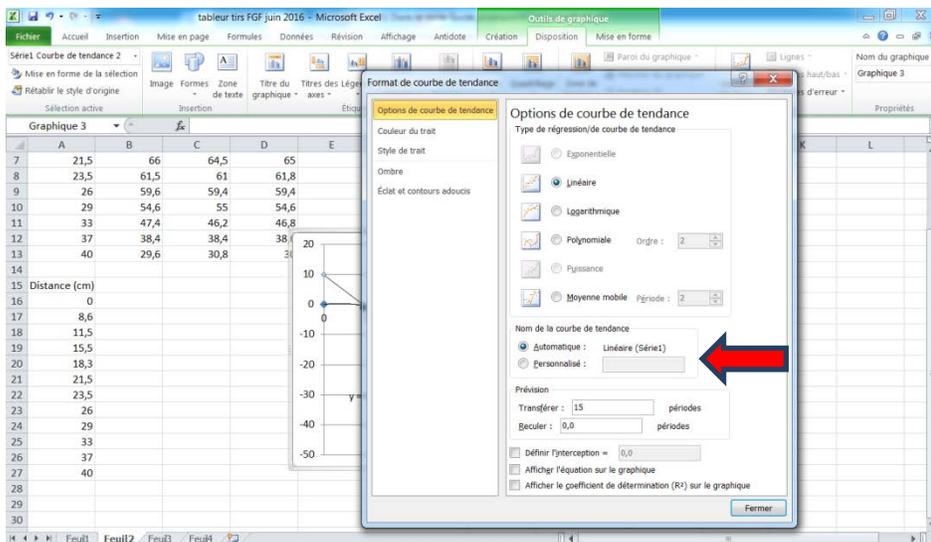
Appendix 4 – Tips for extrapolating with a spreadsheet

Here is a tip to view the long-term trend of your data.

- 1- First, click on one of the data (blue dot) in your graph. Then, from the Layout tab, choose Trendline Options at the bottom of the menu.



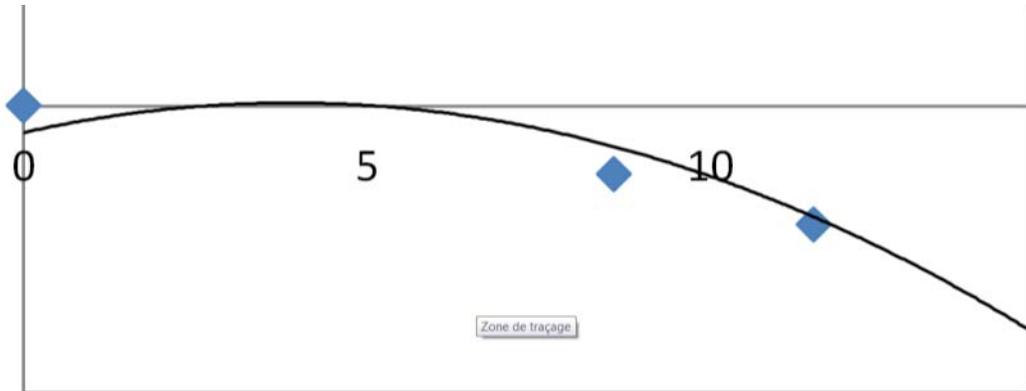
- 2- Then, in the Forward Forecast box, enter 15 or any other value according to your data.



- 3- Click on Close. You can now extrapolate graphically.

Appendix 5 – Tips for using a spreadsheet

Enlarging the graph



When you enlarge the graph, you notice that the peak is not where it should be at (0,0) because it's a trend line. The start point isn't at (0,0) either. Furthermore, it's unlikely that the marble goes up before it goes down in our experiment.

We will force Excel to use (0,0) as the origin. Look at the video (in French) to learn how to proceed:

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If we continue to cheat to obtain the form $y = ax^2$, we get:

$$y = -0.0334x^2 + 0.2256x$$

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$$y = -0.0334x(x - 6.75)$$

If we state that the square root of 6.75 is in fact 0, we get:

$$y = -0.0334x(x - 0.00)$$

$$y = -0.0334x^2$$

And there you have it, the rule for the function is simplified.

Will this simplification have an impact on the accuracy of the algebraic model?
It's up to you to check it and decide which model you want to use to meet the challenge!