My Education Investment

MTH-4151-1

ALGEBRAIC AND GRAPHICAL MODELLING IN A GENERAL CONTEXT

Adult Learners' Workbook

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INFORMATION ABOUT THE SITUATIONAL PROBLEM

Approximate duration: 4–5 hours

Brief Description
You have just been hired by a financial institution for a summer job and your first assignment is to familiarize yourself with the various types of education savings products offered by your institution. After looking at three types of investment, you have to advise a new customer on the type of education savings best suited to his situation.

Targeted Broad Area of Learning
- Environment and consumption: adult learners are encouraged to think about their savings strategies.

Cross-Curricular Competencies
- Exercises critical judgment;
- Uses information;
- Uses information and communications technologies (a spreadsheet is strongly recommended for this LS).

Subject-Specific Competencies
- Use strategies to solve situational problems;
- Uses mathematical reasoning;
- Communicates by using mathematical language.

Integrative Process
- Represents a situation using an algebraic or graphical model;
- Performs interpolation or extrapolation using an algebraic or graphical model.

Knowledge Mobilized
- Relations, functions and reciprocals
  - The actual functions studied are:
    - second degree polynomial function $f(x) = ax^2$
    - exponential function $f(x) = ab^x$ where $a \neq 0$ and $b > 0$

End-of-Course Outcomes

- In order to make decisions, you will need to interpolate or extrapolate results from an algebraic or graphical model. You will also have to interpret the model by making links between message elements and distinguishing between those that are relevant and those that are not.
- Mathematical reasoning stems from generalization using an algebraic model, from a set of situations. To do so, you will need to determine questions according to the observed regularities. You will have to gather relevant information on relationships between

quantities (growth rates of exponential functions, height of tiers—and their length in the case of step functions, etc.).

WORK AND STRATEGY SUGGESTIONS

- Individual or team work;
- Possible integration of ICTs using a spreadsheet;
- If you have difficulty dealing with the situation, see the strategies proposed at the end of this LS for help.

BACKGROUND

You have just been hired by a financial institution for a summer internship and your first assignment is to familiarize yourself with the various types of education savings products offered by your institution. After looking at three types of investment (RESP, TFSA, Learning Bonds), you have to advise a new customer on the type of education savings best suited to his situation. Here is the information provided by your first customer:

- Young student age 16
- $2,500 to invest for post-secondary studies
- Plans to withdraw the education investment in seven years

To help you with your analysis, your internship supervisor has told you that the first two types of investment (RESP and TFSA) offer a return with compound interest of 6% calculated annually. Learning Bonds offer 5% compound interest calculated monthly. He also strongly suggests you read the documentation on these investments (appendices).

Tasks:

Using the problem-solving procedure (4 steps), determine the type of investment that offers the best return for your customer. (Task 1)

Then, analyze an investment strategy to determine its value and prepare a brief summary for your customer explaining your position on the various investment vehicles and strategies using mathematical arguments. (Task 2)

You can also deal with this LS using your own approach. However, you should have your approach validated by your instructor before getting too far into it. If you are unsure where to start, we suggest the following steps.

CARRYING OUT TASKS—TASK 1

Representation

Take a look at the three attached documents and the webography and list the elements that you think are relevant by describing them in the mathematical terms you learned previously in the course. Add the constraints described in the context and any other mathematical elements that may be useful to you. **Present your representation in a clear and detailed manner (list, table, conceptual diagram or other form) to your instructor for a quick validation.** Naturally, this may be improved throughout the learning situation.

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Planning

Based on the gathered information, determine the three algebraic rules that will take into account the best relationship between the constraints and the consequences imposed by the three types of savings options.

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Question 1:

Based on your knowledge of the different function types, do you think the algebraic rules you propose are functions?

Yes □ or No □   Explain your answer using mathematical arguments.

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Activation (N.B.: more than one sheet will be needed for this step):

Determine the representation level you feel is most relevant and represent the expected results for the three investment types. In order to make an informed education investment choice, support your choice with compelling mathematical reasoning. *Do not hesitate to use a spreadsheet to maximize your effectiveness!*

Question 2:

As interest is only deposited into your Education Savings account once or twice a year, depending on the case, you will have the same amount of money over a given period. If we were to represent this representation on a Cartesian plane, with time on the x-axis and education savings on the y-axis, what type of curve would you obtain? Explain your answer.

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Question 3:

If you had to redo this task, would you do anything differently? If yes, explain your approach using a few words or a drawing.

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TASK 2

Reflection

You are now ready to propose the type of investment that would provide the best return to your customer. In the meantime, your supervisor has told you about a new investment strategy:

This consists of depositing the amount in an RRSP (6% return) held by one of your customer's parents. The parent would receive a tax deduction that could in turn be deposited into an RESP in the customer's name in order to qualify for the government grant. Naturally, the parent would return the original amount transferred to the RRSP when your customer needs it.

What do you think of this type of investment strategy? Is it better than your previous choice? Explain your answer using mathematical arguments.

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SUMMARY:

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Review strategies used during this learning situation

From the following list of strategies, check the strategy or strategies used during the various steps of your process. This exercise, will only take a few minutes, will enable you to observe whether the learning situation has enabled you to increase or improve your repertoire of strategies. *The more strategies, the more problems solved!*

Step	Strategies	Used
Representation	Write down the elements of the situation that you think are relevant, then look for a dependency to determine the variables;	☐
	Inventory the strategies to be used and your relevant knowledge of algebra;	☐
	Using sample numbers, estimate the existing types of relationships between the variables of the situation;	☐
	Describe the characteristics of the situation.	☐
Planning	Systematically search for the functional model that is most appropriate to the situation, while keeping in mind this model's accuracy limitations;	☐
	Look for an algebraic rule that will take into account the best relationship between the constraints to be respected and the consequences imposed by the situational problem.	☐
Activation	Make a simulation using concrete objects or technology to determine a relationship;	☐
	Use technology (spreadsheets, graphing calculators, etc.) to analyze the role of a function's various parameters;	☐
	Using the parameters of a function, make a sketch to predict results.	☐
Reflection	Compare your results with the expected results or the results obtained by others;	☐
	Verify the consistency of your solution by making sure that the values found respect the image of the function, for example;	☐
	Use a metacognitive questions grid, (for example: Why am I using this approach? What would I change, and why?);	☐
	Use a calculator to validate your work.	☐

Turning the LS into an ES…

You will find on the following page a list of observable elements to help you determine whether (or not) you have reached the level of mathematical skills development required for this course. We recommend that the instructor coach you during the correction.

2017-12-26

MATHEMATICAL SKILLS SELF-EVALUATION GRID

Competency 1 *Uses strategies to solve situational problems*			
Criteria chosen for the evaluation	**Observable evaluation criteria indications**	**Yes**	**No**
1.1 Indication (oral or written) that the situational problem has been understood	*Written expression:* • I identify what is being sought. • I uncover useful data. • I take constraints into account. **OR** • I *highlight* what is sought in one colour. • I *highlight* useful data in another colour. • I *highlight* the constraints using another colour. Other: -	☐ ☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐ ☐ ☐ ☐
1.2 Application of strategies and appropriate mathematical knowledge	• I choose the required processes and steps (mathematical approach). • I choose the types of representation that will allow me to make the case for my savings choice. • I make connections between the context and associated representation. • I question my initial approach in order to improve it (if necessary).	☐ ☐ ☐ ☐	☐ ☐ ☐ ☐

Competency 2 *Uses mathematical reasoning*			
Criteria chosen for the evaluation	**Observable evaluation criteria indications**	**Yes**	**No**
2.1 Correct use of appropriate mathematical concepts and processes	I use knowledge appropriate to the task: ▪ second degree polynomial function $f(x)= ax_2$ ▪ exponential function $f(x)=ab_x$ where $a \neq 0$ and $b > 0$ I develop integrative processes appropriate to the task: • Represents a situation using an algebraic or graphical model; • Performs interpolation or extrapolation using an algebraic or graphical model.	☐ ☐ ☐ ☐	☐ ☐ ☐ ☐
2.2 Proper implementation of mathematical reasoning suited to the situation	• I present a consistent (logical) approach to solve the situational problem (task).	☐	☐
2.4 Proper organization of the steps in an appropriate procedure	• I structure my approach to present clear steps that comply with mathematical rules and conventions.	☐	☐

WEBOGRAPHY

1. http://www.epq.gouv.qc.ca/A/Info/produits_offerts/celi.aspx, Épargne Placement Québec web page on TFSAs, consulted on June 10, 2016.
2. http://www.reee.ca/quest-ce-quun-reee.html, non-government web site by financial advisor Maud Salomon, consulted on June 10, 2016 [French only].
3. https://www.canada.ca/en/revenue-agency/services/tax/individuals/topics/registered-education-savings-plans-resps.html, Canada Revenue Agency web site, consulted on June 10, 2016.
4. http://www.reee.ca/reee-celi-ou-reer.html, non-government web site by financial advisor Maud Salomon, consulted on June 10, 2016 [French only].
5. http://www.revenuquebec.ca/en/citoyen/situation/parent/autres_infos/iqee/default.aspx, Revenu Québec web site, consulted on June 10, 2016.
6. https://www.canada.ca/en/employment-social-development/services/student-financial-aid.html, Government of Canada web site on education planning, consulted on June 10, 2016.
7. https://www.getsmarteraboutmoney.ca/calculators/resp-savings-calculator/, RESP savings calculator on the Ontario Securities Commission's web site, consulted on June 10, 2016.

Wikipedia compound interest calculation formula

$$V_f = V_i \cdot (1 + \rho)^a$$

where V_f is the final value, V_i is the initial value, ρ is the interest rate over a period, and a is the number of periods (years, semesters, quarters, months, etc.). The interest rate is usually expressed as a percentage, thus we would write 2% for ρ = 0, 02

N.B.: This does not take into account the QESI, CLB (child born after 2003) and other credits as these are tax credits for parents and do not apply to adult investors.

Appendix on RESP—Registered Education Savings Plan

http://www.reee.ca/quest-ce-quun-reee.html

An RESP is a registered education savings plan by the Canada Revenue Agency (CRA) that provides certain tax benefits. It helps families save for their children's post-secondary education. Contributions grow tax-free until they are withdrawn to pay for the education of the child (beneficiary) at a designated post-secondary institution.

One of the unique advantages offered by RESPs is the fact that the government of Canada contributes 20% of the amount paid into an eligible beneficiary's RESP to a maximum of $500 a year and up to a lifetime maximum of $7,200 per beneficiary. Other incentives may be added to these contributions, depending on where you live and your family's net annual income. Beneficiaries who have reached the age of 16 or 17 must meet certain criteria to be eligible for the government grant.

An RESP allows your investment to grow tax sheltered and provides an opportunity to distribute income. Funds will be taxed on withdrawal at the recipient's tax rate, who will generally pay little tax. The tax payable (if any) will thus be minimal. Only the interest earned on these subsidies and on the principal are taxable. The subscriber may make principal withdrawals without incurring any tax consequences, as the contributions had been made in after-tax dollars. While the beneficiary is registered at a post-secondary educational institution (and six months thereafter), plan contributions may be withdrawn without any tax consequences on government subsidies made under the plan.

https://www.canada.ca/en/revenue-agency/services/tax/individuals/topics/registered-education-savings-plans-resps.html

Registered Education Savings Plan (RESP)

A Registered Education Savings Plan (RESP) is a contract between an individual (the subscriber) and a person or organization (the promoter).

Under the contract, the subscriber names one or more beneficiaries (the future student[s]) and agrees to make contributions for them, and the promoter agrees to pay educational assistance payments (EAPs) to the beneficiaries.

There are two different types of RESP available: family plans and specified plans. The RESP rate of return is currently 6%[1].

http://www.servicecanada.gc.ca/fra/gdc/scee.shtml

Canada Education Savings Grant (CESG)

The Canada Education Savings Grant is money the Government of Canada adds to your child's Registered Education Savings Plan (RESP).

The grant is comprised of two parts:

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[1] The return is presented for information purposes only. Please contact us to find out the available rates of return.

- The basic Canada Education Savings Grant (CESG)
- No matter what your family income is, the CESG pays a basic of 20% of annual contributions you make to your child's RESP, on contributions of up to $2,500 a year.
- Additional CESG Depending on your net family income, you could receive an additional 10% or 20% on the first $500 contributed to your child's RESP each year.

Administered by: Employment and Social Development Canada (ESDC)

Information on eligibility

Who is eligible for this grant?

- Beneficiaries qualify for a grant on the contributions made on their behalf up to the end of the calendar year in which they turn 17 years of age; they must be Canadian residents and beneficiaries of an RESP.
- Specific rules apply to beneficiaries who are 15 to 17 years of age. See the page Rules for Children 15 to 17 Years of Age for more information.

Application information

- Go to the CanLearn web site for more information.
- Information on RESPs and the CESG application process are offered by financial institutions such as banks or caisses populaires as well as by group plan advisors and certified financial planners. These institutions, advisors and planners are called "RESP providers."

Financial information

The lifetime maximum grant payable by the government to your child's RESP is $7,200.

Your child may use these funds for full-time or part-time studies in a vocational program, a CEGEP, a college, trade school or university.

Contact

- General information: 1 800 O-Canada (1-800-622-6232)

TTY: 1-800-926-9105

N.B.: This does not take into account the QESI, CLB (child born after 2003) and other credits as these are tax credits for parents and do not apply to adult investors.

Appendix on Learning Bonds—Learning Bonds

Learning Bonds are available now!

For individuals 18 years of age or more with no other income.

When you buy Learning Bonds, your principal is fully guaranteed and you **receive interest calculated and deposited into your Learning Bonds account twice a year**.

• Guaranteed 5% return on maturity.
• Choice of five-year, ten-year or longer terms.
• Payable upon maturity of the selected term.
• More frequent periodic payments than RESPs or TFSAs.
• Minimum purchase of $100.

Ready to take action? We invite you to speak to an investment officer by calling toll-free.

1-800-xxx-xxxx

Monday to Friday, 8 a.m. to 8 p.m.