Just you wait and see what I can do!

The Case of the Emerald Ash Borer in Québec

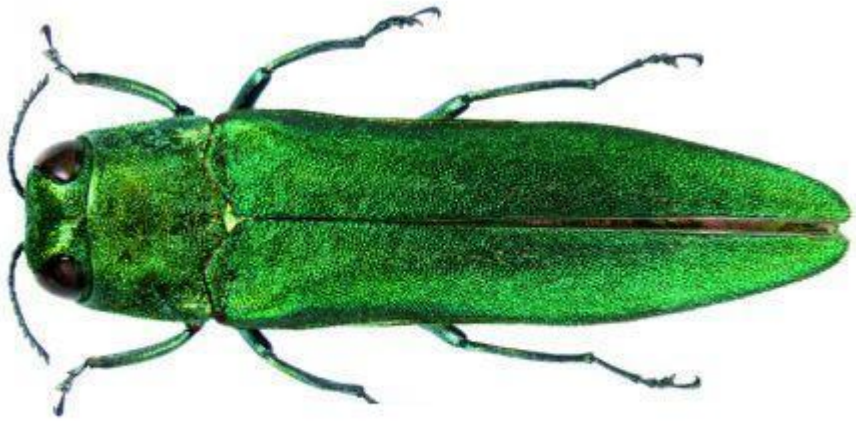


Photo : © Klaus Bolte, CFS-SCF, NRCan-RNCan [1]

MTH-4151-1

ALGEBRAIC AND GRAPHICAL MODELLING IN THE GENERAL CONTEXT

NAME : _____ DATE : _____

Louise Roy

11 January 2018

Attention!

The use of a spreadsheet is strongly recommended for this LS

[1] Source: Environment and Climate Change Canada : http://www.ec.gc.ca/biotrousses-biokits/default.asp?lang=En&n=F43C77ED-1, copy of an official document from the Government of Canada, non-commercial reproduction

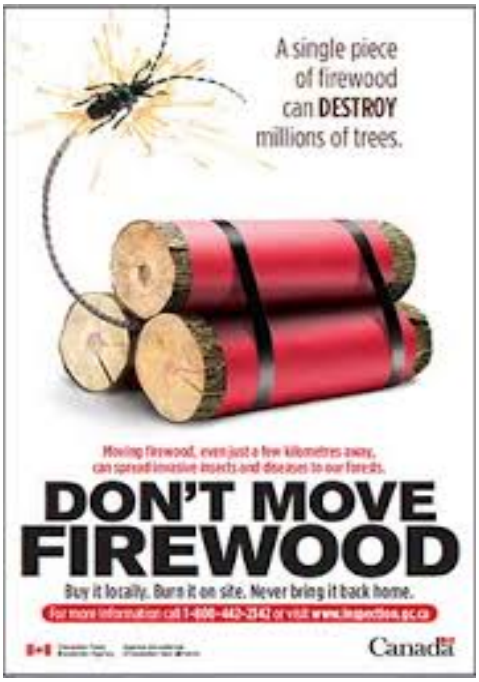
Context

Camp grounds in federal and provincial parks have a regulation that prohibits wood being brought in for camp fires. Wood must be purchased on site. [2]

Do you know why?

In fact, the reason why is the presence of numerous exotic insects that attack trees, including a little insect originally from Asia. Present in Canada since 2002, it has been found in the Montréal région since 2011. Known as the **Emerald Ash Borer,** this **insect** attacks **ash** trees.

Transporting logs from one region to another increases the risk of the insect being spread across Québec. But how, as featured on the poster opposite, can **a single piece of firewood destroy millions of trees?** And once these trees are destroyed, what are the consequences for a forestry producer?



A Situational Problem

To understand the scale of damage caused by an insect infestation, you will simulate the loss in value of a small wood lot infested with emerald ash borers transported in by just a few individuals. The aim is to demonstrate the impact of an infestation to convince campers **not to bring in their own firewood**.

To do this, you will complete three tasks:

- A graphical model of the growth of an emerald ash borer population;
- Demonstration of the impact of an infestation on a wood lot;
- Estimatation of the damage and the cost of treatments.

[2] Source: http://publications.gc.ca/site/eng/330263/publication.html, May 2013

The Situational Problem

You are to demonstrate the impact of just a few emerald ash borers on a forest to convince people not to bring in their own firewood when they go camping.

The data needed to complete this situational problem can be found at the end of this document.

Task 1: A graphical model of the growth of the emerald ash borer poulation

For this task, you will simulate a situation where there is an unlimited abundance of resources for an insect with no natural enemies and where all the eggs (survival rate of 100%) produce adults in the following year[3].

Complete the following table (the use of a spreadsheet is recommended):

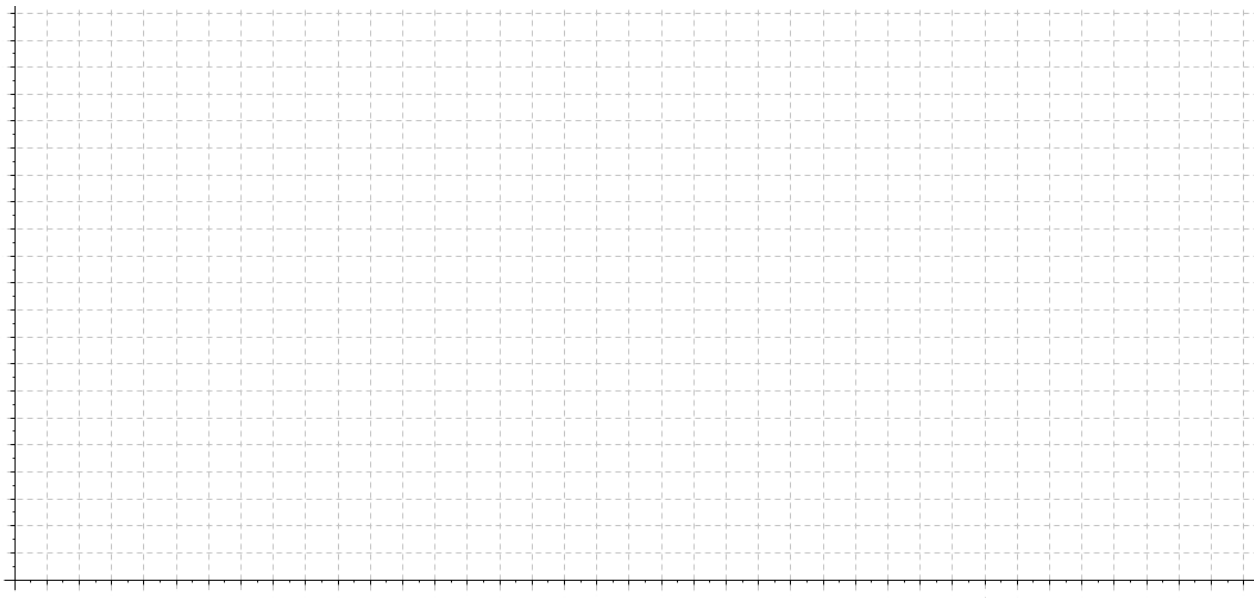
Average number of eggs laid by a female: _____

Year	Number of Insects
1	2 (*1 male + 1 female*)
2	
3	
4	
5	

Explain how you obtained your results:

[3] Use the average number of eggs laid by a female. (Don't forget, only the females lay eggs).

With the help of a spreadsheet, draw the graph showing the number of insects per year. Print your graph or reproduce it on the graph below.



Task 2: Demonstration of the impact of an infestation on a wood lot

For this task, you will predict how much damage would occur in a particular area of the forest.

Let us assume that it only takes 500 larvae to kill one tree and the infested tree takes about two years to die. Create an algebraic model that estimates the time from the introduction of a non-native species to when symptoms first start to appear.

This model should allow you to predict how many years it will take for all ash trees to die.

Example

A wood lot covering an area (A) of 30 hectares has an average density (D) of 1 000 trunks per hectare. It is estimated that ash trees make up 1% of the total percentage (P) of trees on this lot.

Create an algebraic model to calculate the number (N) of ash trees.

How many ash trees are there on this lot?

How many trees less or more (interval) a margin of error of 0.1 % is the percentage of ash trees equivalent to?

If a couple of emerald ash borers infest this wood lot, how many years will it be before the owner begins to see the ash trees die? Use the results obtained in Task 1.

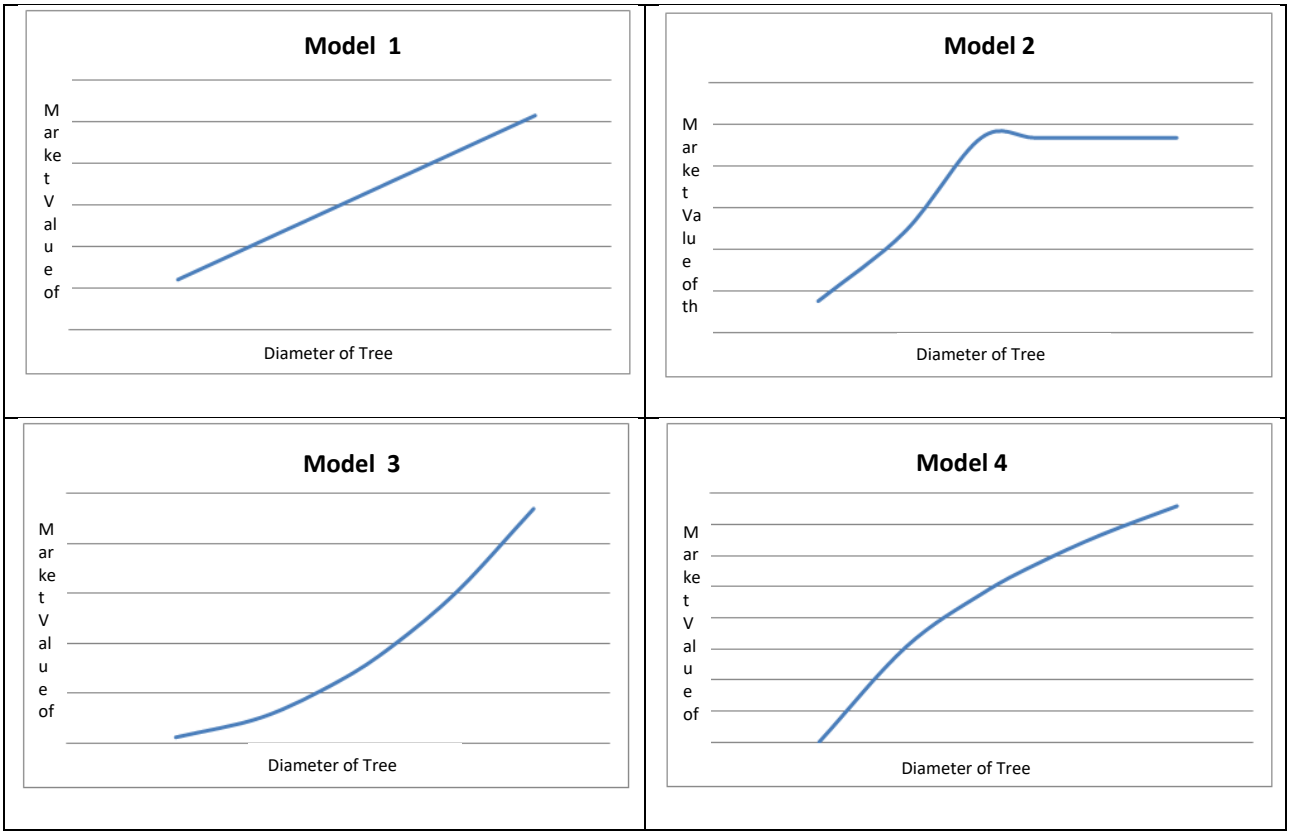
If the trees are not treated, how many years will it be before all the trees die?

Task 3: Estimation of the damage and an assessment of the treatment

For this task, you will estimate the financial losses incurred by the owner of the infested wood lot.

The value of a tree destined for the forest industry is calculated according to the **volume of useable timber** ($/m^3).

Which of the following graphs best represents the relationship between the diameter of a tree and its market value?

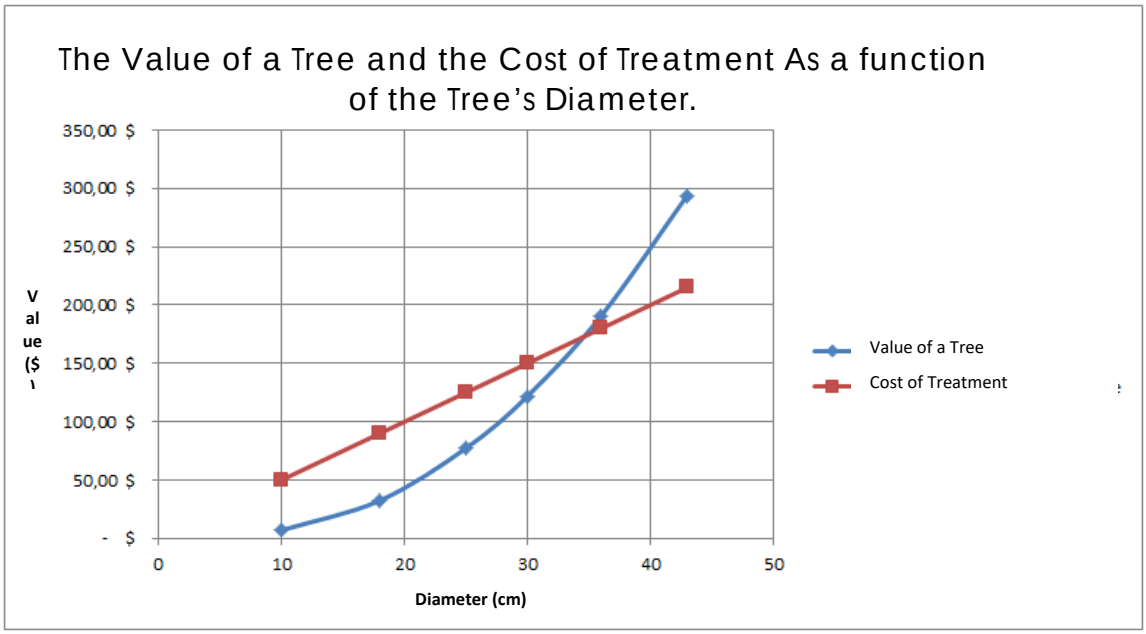


Explain your choice:

And if the trees are treated?

A treatment is available and can save the trees right at the start of the infestation. The cost of the treatment is based on the diameter of the tree.

Here is a graph showing the value of the tree and the cost of the treatment based on its diameter:



In what type of forest would the treatment be more cost efficient than destroying the trees?

a) In a young forest;
b) In a mature forest;
c) The treatment is always preferable to destroying trees;
d) Destroying trees is always more cost efficient than treating them.

Explain your answer:

16-10-2012 - Arrêt des livraisons de frêne à Thurso

Prenez note qu'à compter du lundi 5 novembre 2012, l'usine Fortress Cellulose Spécialisée n'acceptera plus de tronçons de frêne venant du territoire du Syndicat du Sud-Ouest en raison de l'infestation d'agrile du frêne qui s'étend de plus en plus.

01-10-2012 - Rappel Agrile du frêne

Un petit rappel concernant l'agrile du frêne. Aucune livraison de frêne n'est acceptée en provenance des municipalités suivantes : Chelsea, l'Ange-Gardien, Gatineau, Carignan, Chambly, Richelieu, Saint-Basile-le-Grand, Saint-Mathias-sur-Richelieu, Laval, Longueuil ainsi que tout le territoire de la ville de Montréal. Pour de plus amples informations, veuillez visiter le site de l'agence canadienne d'inspection des aliments.

Paper mills are no longer buying ash trees! What could be a possible consequence of this decision?

Source (screen capture): http://www.syndicatbois.com/, May, 2013

Cultural references[4]

The study of population growth is not new. One of the best known models was put forward around 1798 by the British economist, *Robert Malthus* (1766-1834), who was primarily interested in human populations. He demonstrated that, sooner or later, a population would run short of resources because populations grew exponentially and available resources increased linearly. According to *Charles Darwin (1809-1882),* natural selection favoured those individuals best adapted to their surroundings.

During the same era, the Belgian mathematician *Pierre-François Verhulst* (1804-1849) explained the dynamics of animal populations. He developed a model, known as the *logistic function,* that not only took into account the animal's environment but also the constraints that affect animal population growth.

Today, numerous other models try to explain and predict the growth of various animal, plant and micro-organism populations by integrating different factors into the model such as available resources, the ratio of prey-to-predator, natural resource withdrawals, parasites, etc.

Reflection and analysis

The following questions will help deepen your understanding of algebraic modeling and its relevance to the study of populations.

a) Knowing that a model will never be perfect, what's the point of making a model to explain the evolution of a population?

b) How can a model be designed to reflect reality as closely as possible? What would be the consequences of this from a mathematical perspective?

c) If you had to create a model to estimate the evolution of a human population (in a city or a country), what variables should you take into consideration?

[4] Sources: http://www.aestq.org/sautquantique/telechargement/chaos_mathematica.pdf and Wikipedia

Useful data[5]

What is known about these insects

- The life cycle of the insect is generally a little more than a year, with only the larval stage surviving the winter;
- Each female lays an average of 76 eggs;
- A female can lay up to 275 eggs;
- Several females can lay their eggs in the same tree;
- A female can lay her eggs in several trees;
- The larvae kill the trees;
- An infested tree dies after two or three years;
- In North America, the estimated cost of the damage in urban settings alone is 12 billion dollars;
- In general, five years after the onset of an infestation, 98 % of the ash trees in a region will have been killed by the insect;
- Some parasites kill a certain percentage of eggs;
- Woodpeckers find and eat a certain percentage of larvae;
- When tree density is low, the adults can move within a radius of 10km from their tree of birth;
- A treatment costs between $5,00 and $7,00 per centimetre of the tree's diameter. The trees must be treated every two years for as long as these insects are found in the surrounding area.

Assumptions

- It can be assumed that half of the eggs produce females;
- The number of larvae needed to kill the tree will depend on its size.

The variables (the unknowns)

Initial hypotheses: establish an interval in which lie the value of the variables.

- The percentage of eggs and larvae that survive;
- The minimum number of larvae it takes to kill a tree;
- The maximum number of larvae for which treatment is effective;
- The percentage of eggs killed by parasites;
- The percentage of larvae eaten by woodpeckers;
- The percentage of females that lay according to the density and the proximity of trees.

[5] Only certain data will be useful for this situational problem. The rest of the information can be used for the exercises aimed at deepening your understanding.

Simulation of a wood lot

These data give you an order of magnitude of possible values. These figures vary greatly depending on the region and the year. They are only possible values not real values.

Area in hectares (ha)	between 20 and 100 ha
Trunk density per hectare	between 1500 and 2 000
Percentage of ash trees	between 0.2 % and 3 %
Average diameter of ash trees	between 25 and 35 cm
Average volume of ash trees [6]	Between 1 and 2.4 m^3
Price of ash	between \$55 and \$80 per m^3

[6] Be careful, the volume is based on the diameter and height of the tree.